Iowa State University
ECpE Department

Solving Power Flow Using Bus Injection Model

Acknowledgement

[1] Kersting, William H. "Distribution system modeling and analysis." Electric power generation, transmission, and distribution. CRC press, 2018. 26-1.

[2] McCalley, James D. "The Power Flow Problem." https://home.engineering.iastate.edu/~jdm/ee553/PowerFlow.pdf.

[3] Dugan, Roger C., and T. McDermott. "Reference guide." The Open Distribution System Simulator (OpenDSS). EPRI (2016).

[4] "Yprimitive and Ymatrix." YouTube, https://youtu.be/TRfcX6_D3Fw?si=bVURan_lGfx1JuzF.

1 Overall Structure of Solving Power Flow

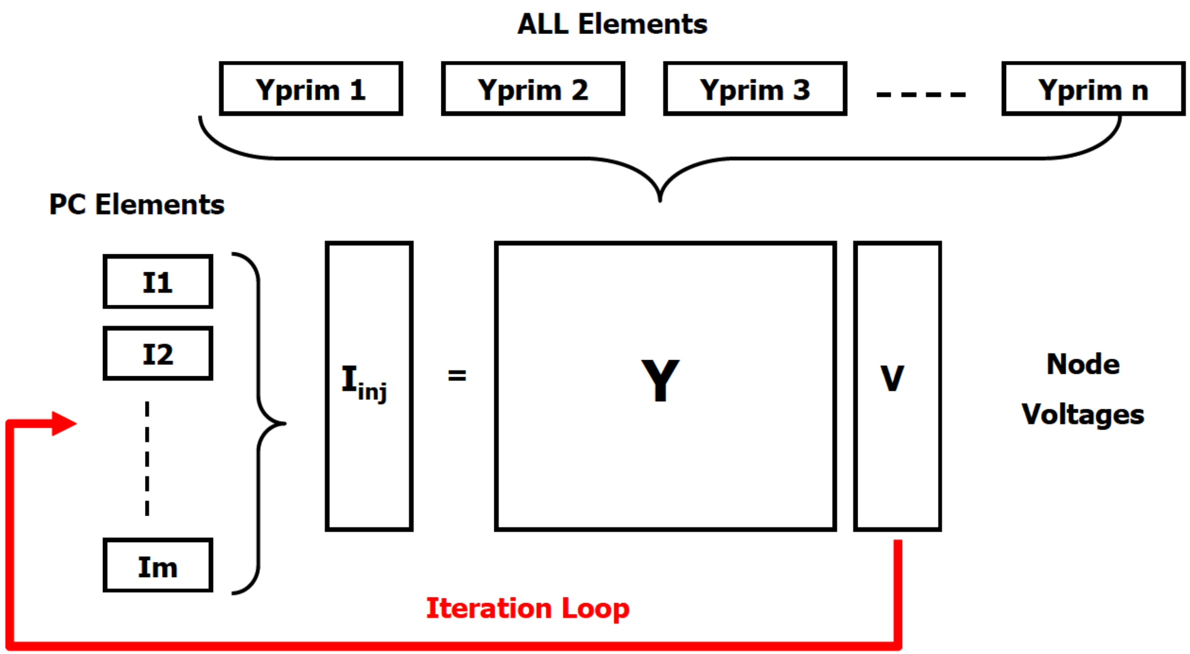
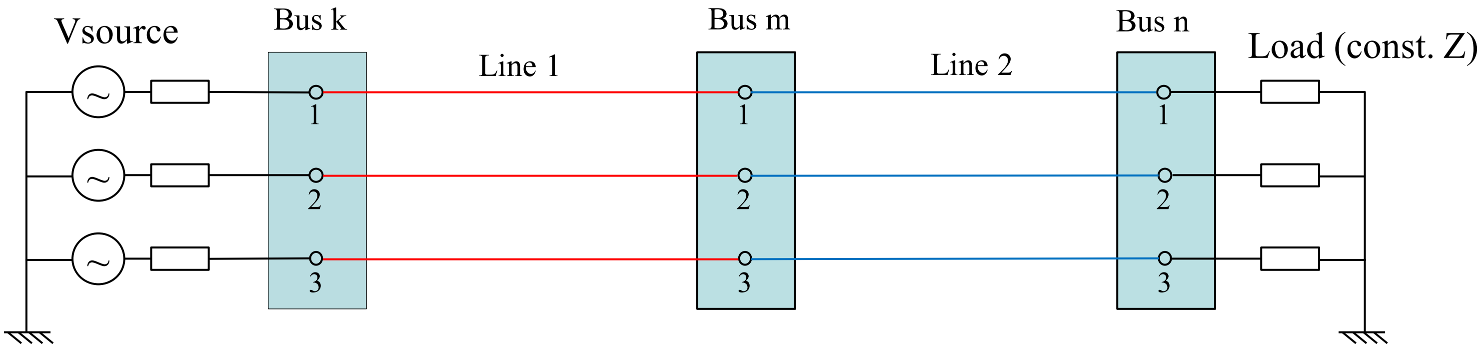


Figure 17. OpenDSS Solution Loop

This figure illustrates how the OpenDSS puts all the *power delivery* elements and *power conversion* elements together to perform a solution.

- Power delivery elements *transport* energy from one point to another.
  - Line, transformer, capacitor and reactor, etc.
- Power conversion elements *convert* power from electrical form to some other form, or vice versa.
  - Load and generator, etc.

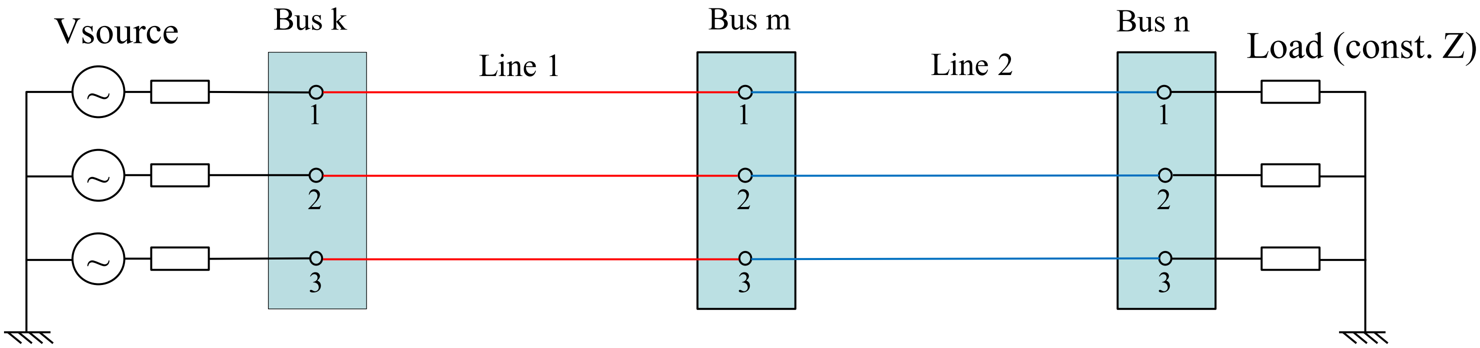
2 Example 1: Constant-Z Load



OpenDSS model:

```
Clear
//-------------- Define a new circuit --------------------//
new circuit.experiment

//---------------Define a Voltage source------------------//
Edit "Vsource.source" phases=3 basekv=13.8 Pu=1.02 bus1=K R1=5 X1=0 R0=5 X0=0

//----------------- Define line codes  --------------------//
New LineCode.Code1 nphases= 3 Units= mi
 ~ Rmatrix= (0.62 | 0.17 0.62 | 0.17 0.17 0.62 )
 ~ Xmatrix= (1.21 | 0.43 1.21 | 0.43 0.43 1.21 )

New LineCode.Code2 nphases= 3 Units= mi
 ~ Rmatrix= (1.19 | 0.23 1.19 | 0.23 0.23 1.19 )
 ~ Xmatrix= (1.21 | 0.49 1.21 | 0.49 0.49 1.21 )
… (continued)
```

2 Example 1: Constant-Z Load

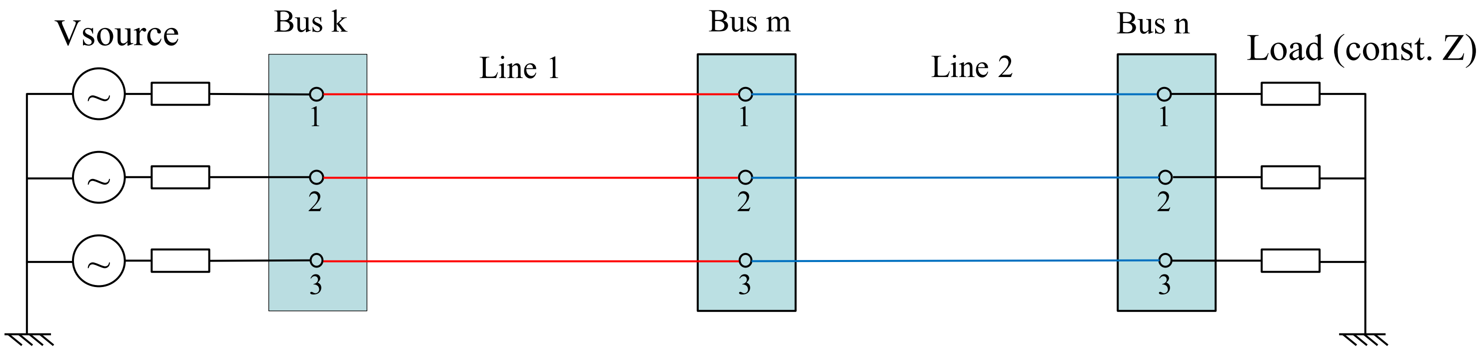


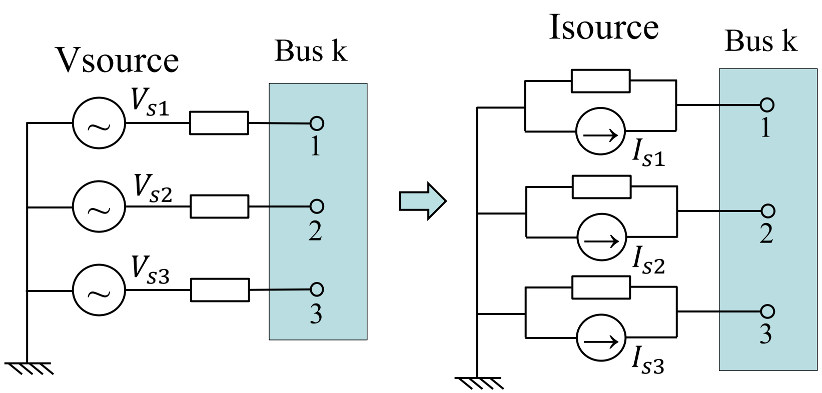
OpenDSS model:

```
… (continued)
//----------------  Define line 1 and 2  ------------------//
New Line.1 phases=3  Bus1=k.1.2.3 Bus2=m.1.2.3  length=1 units=mi LineCode=Code1
New Line.2 phases=3  Bus1=m.1.2.3 Bus2=n.1.2.3  length=1 units=mi LineCode=Code2

//----------------  Define a load  ------------------------//
New Load.L Bus1=n.1.2.3.0       Phases=3 Conn=wye Model=2 kV=13.8   kW=500 kvar=0

Set VoltageBases = "13.8"
Solve
```

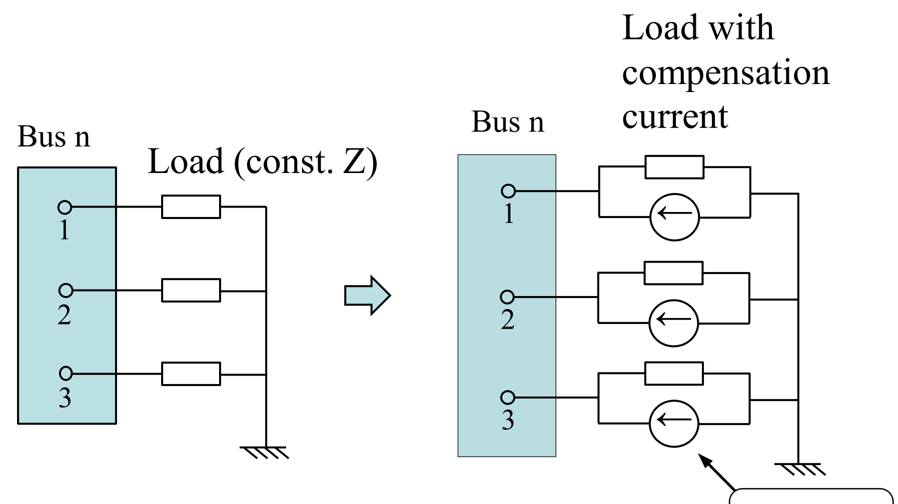
Constant-impedance load

2 Example 1: Constant-Z Load



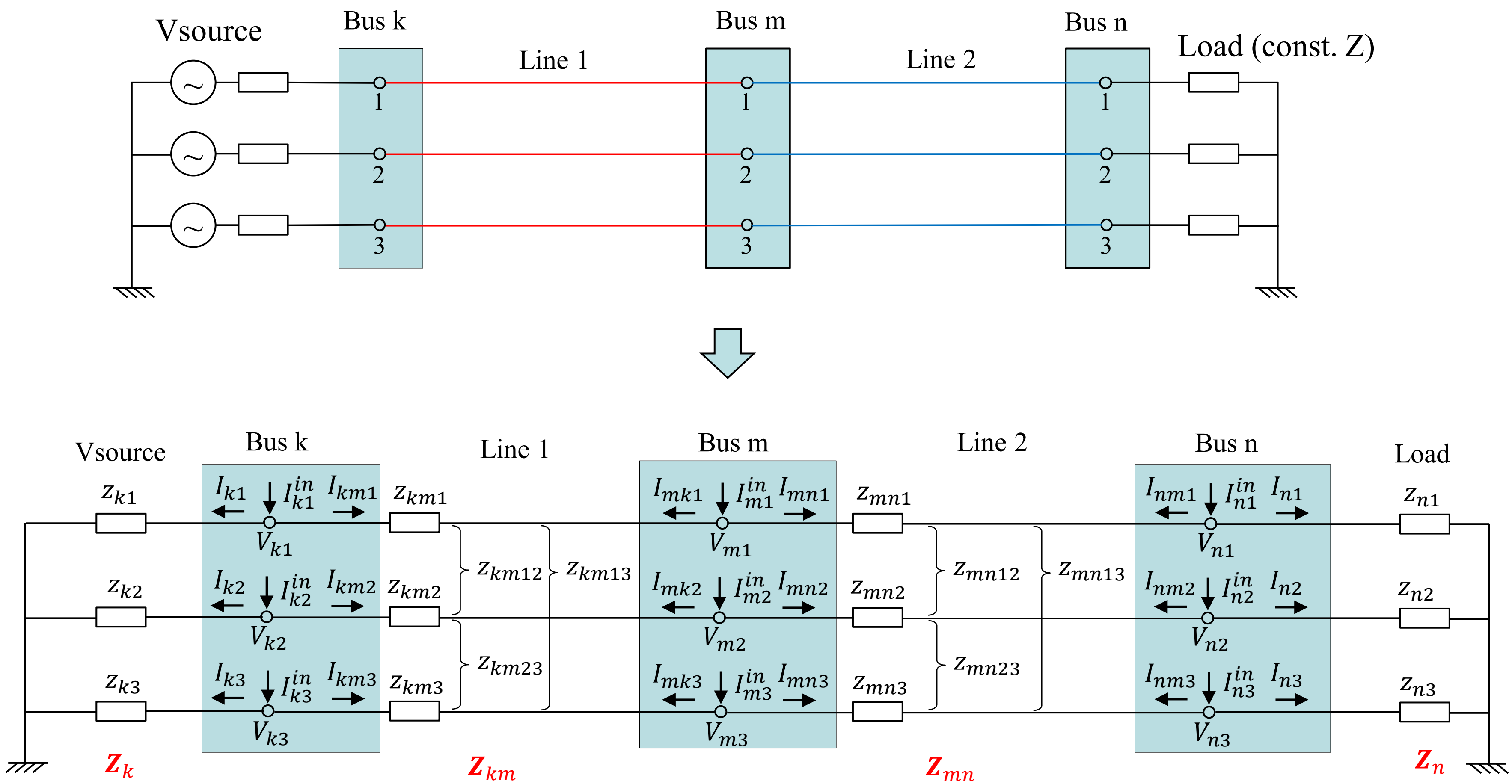


Thevinen equivalent to Norton equivalent



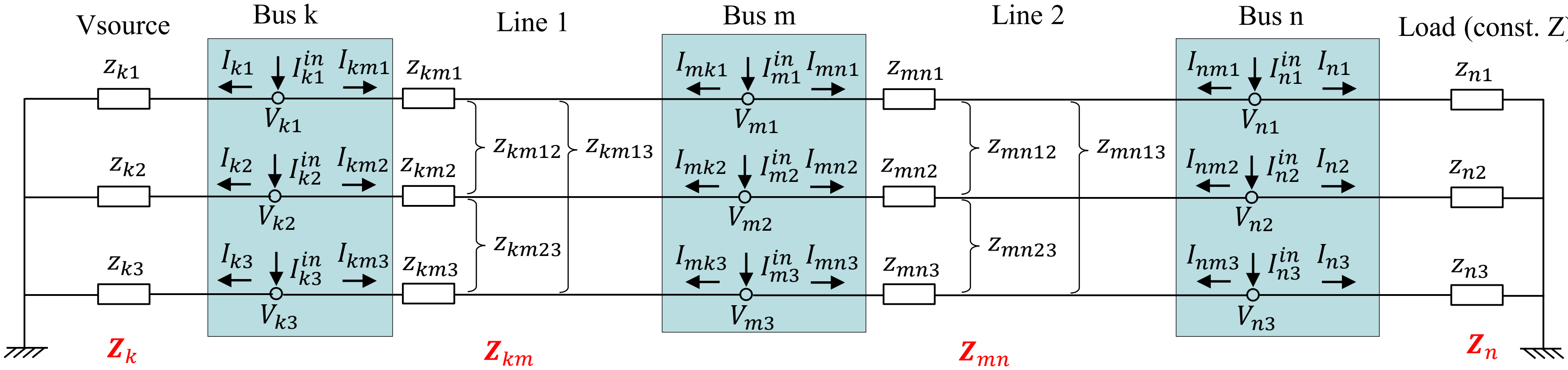
Various loads to a generic load model
(Constant Z + compensation current)

2 Example 1: Constant-Z Load



$I_{k1}^{in} = I_{s1} = \frac{V_{s1}}{z_{k1}}, I_{k2}^{in} = I_{s2} = \frac{V_{s2}}{z_{k2}}, I_{k3}^{in} = I_{s3} = \frac{V_{s3}}{z_{k3}}$ $\quad I_{m1}^{in} = I_{m2}^{in} = I_{m3}^{in} = 0$ $\quad I_{n1}^{in} = I_{n2}^{in} = I_{n3}^{in} = 0$

2 Example 1: Constant-Z Load



Definitions:

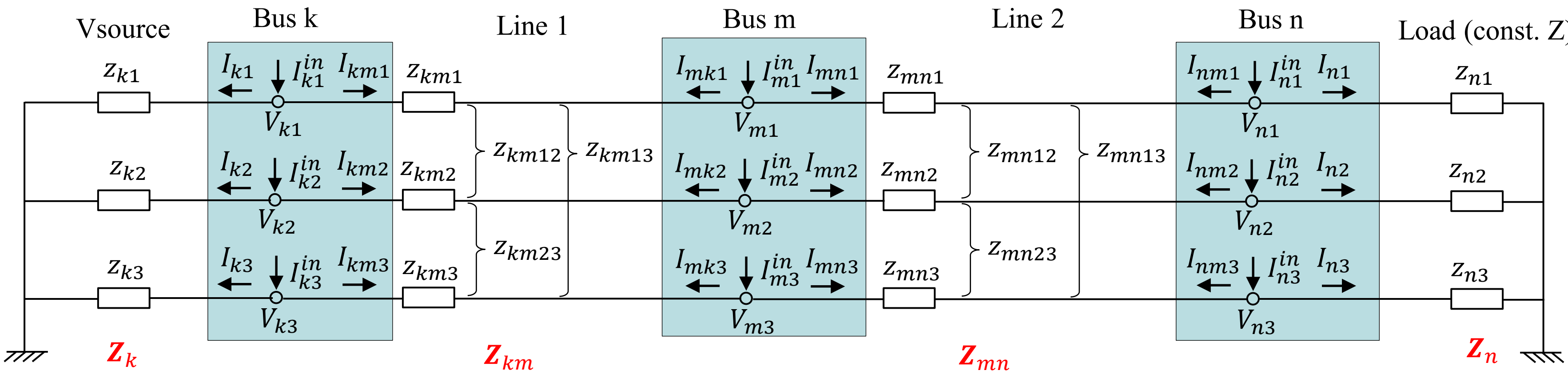
$$\boldsymbol{Z}_k = \begin{bmatrix} z_{k1} & 0 & 0 \\ 0 & z_{k2} & 0 \\ 0 & 0 & z_{k3} \end{bmatrix}, \quad \boldsymbol{Y}_k = \boldsymbol{Z}_k^{-1} = \begin{bmatrix} y_{k1} & 0 & 0 \\ 0 & y_{k2} & 0 \\ 0 & 0 & y_{k3} \end{bmatrix}$$

$$\boldsymbol{Z}_{km} = \begin{bmatrix} z_{km1} & z_{km12} & z_{km13} \\ z_{km12} & z_{km2} & z_{km23} \\ z_{km13} & z_{km23} & z_{km3} \end{bmatrix}, \quad \boldsymbol{Y}_{km} = \boldsymbol{Z}_{km}^{-1} = \begin{bmatrix} y_{km1} & y_{km12} & y_{km13} \\ y_{km12} & y_{km2} & y_{km23} \\ y_{km13} & y_{km23} & y_{km3} \end{bmatrix}$$

$$\boldsymbol{Z}_{mn} = \begin{bmatrix} z_{mn1} & z_{mn12} & z_{mn13} \\ z_{mn12} & z_{mn2} & z_{mn23} \\ z_{mn13} & z_{mn23} & z_{mn3} \end{bmatrix}, \quad \boldsymbol{Y}_{mn} = \boldsymbol{Z}_{mn}^{-1} = \begin{bmatrix} y_{mn1} & y_{mn12} & y_{mn13} \\ y_{mn12} & y_{mn2} & y_{mn23} \\ y_{mn13} & y_{mn23} & y_{mn3} \end{bmatrix}$$

$$\boldsymbol{Z}_n = \begin{bmatrix} z_{n1} & 0 & 0 \\ 0 & z_{n2} & 0 \\ 0 & 0 & z_{n3} \end{bmatrix}, \quad \boldsymbol{Y}_n = \boldsymbol{Z}_n^{-1} = \begin{bmatrix} y_{n1} & 0 & 0 \\ 0 & y_{n2} & 0 \\ 0 & 0 & y_{n3} \end{bmatrix}$$

2 Example 1: Constant-Z Load



(1) Bus k:

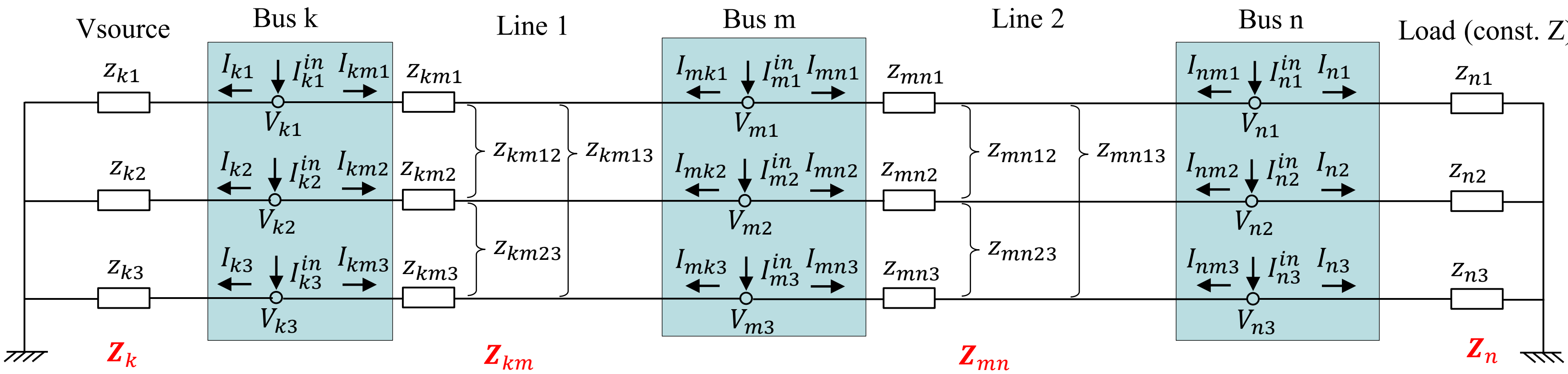
1) Branch currents of Vsource:

According to Kirchhoff's voltage law:

$$\begin{bmatrix} V_{k1} \\ V_{k2} \\ V_{k3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} z_{k1} & 0 & 0 \\ 0 & z_{k2} & 0 \\ 0 & 0 & z_{k3} \end{bmatrix} \begin{bmatrix} I_{k1} \\ I_{k2} \\ I_{k3} \end{bmatrix} \Rightarrow \begin{bmatrix} I_{k1} \\ I_{k2} \\ I_{k3} \end{bmatrix} = \begin{bmatrix} z_{k1} & 0 & 0 \\ 0 & z_{k2} & 0 \\ 0 & 0 & z_{k3} \end{bmatrix}^{-1} \begin{bmatrix} V_{k1} - 0 \\ V_{k2} - 0 \\ V_{k3} - 0 \end{bmatrix} = \begin{bmatrix} y_{k1} & 0 & 0 \\ 0 & y_{k2} & 0 \\ 0 & 0 & y_{k3} \end{bmatrix} \begin{bmatrix} V_{k1} \\ V_{k2} \\ V_{k3} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{I}_k = \boldsymbol{Y}_k \boldsymbol{V}_k, \text{ where, } \boldsymbol{I}_k = \begin{bmatrix} I_{k1} \\ I_{k2} \\ I_{k3} \end{bmatrix}, \boldsymbol{V}_k = \begin{bmatrix} V_{k1} \\ V_{k2} \\ V_{k3} \end{bmatrix}.$$

2 Example 1: Constant-Z Load



(1) Bus k:

2) Branch currents of Line 1:

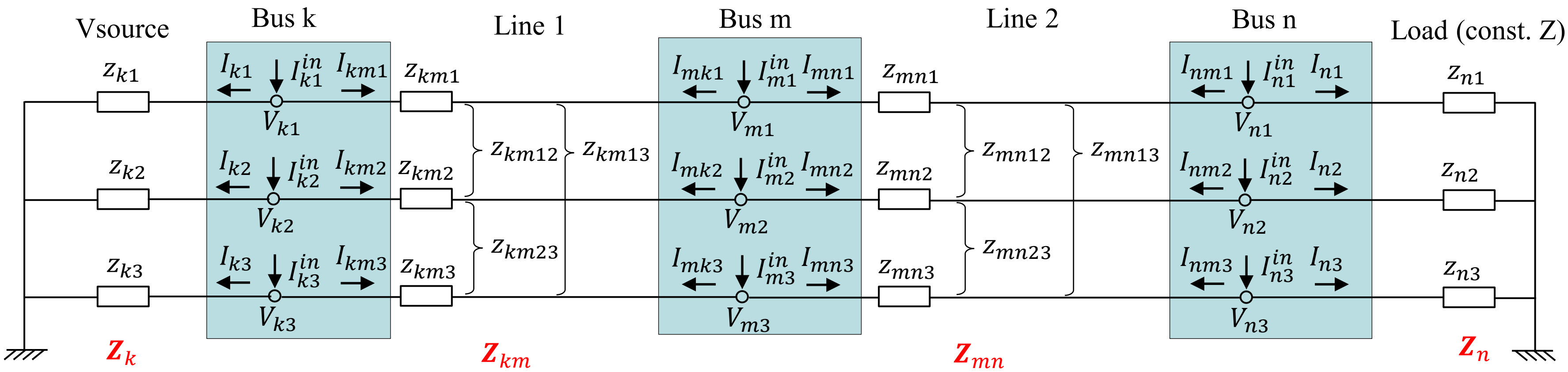
According to Kirchhoff's voltage law:

$$\begin{bmatrix} V_{k1} \\ V_{k2} \\ V_{k3} \end{bmatrix} = \begin{bmatrix} V_{m1} \\ V_{m2} \\ V_{m3} \end{bmatrix} + \begin{bmatrix} z_{km1} & z_{km12} & z_{km13} \\ z_{km12} & z_{km2} & z_{km23} \\ z_{km13} & z_{km23} & z_{km3} \end{bmatrix} \begin{bmatrix} I_{km1} \\ I_{km2} \\ I_{km3} \end{bmatrix} \Rightarrow \begin{bmatrix} I_{km1} \\ I_{km2} \\ I_{km3} \end{bmatrix} = \begin{bmatrix} z_{km1} & z_{km12} & z_{km13} \\ z_{km12} & z_{km2} & z_{km23} \\ z_{km13} & z_{km23} & z_{km3} \end{bmatrix}^{-1} \begin{bmatrix} V_{k1} - V_{m1} \\ V_{k2} - V_{m2} \\ V_{k3} - V_{m3} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} I_{km1} \\ I_{km2} \\ I_{km3} \end{bmatrix} = \begin{bmatrix} y_{km1} & y_{km12} & y_{km13} \\ y_{km12} & y_{km2} & y_{km23} \\ y_{km13} & y_{km23} & y_{km3} \end{bmatrix} \begin{bmatrix} V_{k1} - V_{m1} \\ V_{k2} - V_{m2} \\ V_{k3} - V_{m3} \end{bmatrix} \Rightarrow \boldsymbol{I}_{km} = \boldsymbol{Y}_{km}(\boldsymbol{V}_k - \boldsymbol{V}_m)$$

where, $\boldsymbol{I}_{km} = \begin{bmatrix} I_{km1} \\ I_{km2} \\ I_{km3} \end{bmatrix}, \boldsymbol{V}_m = \begin{bmatrix} V_{m1} \\ V_{m2} \\ V_{m3} \end{bmatrix}.$

2 Example 1: Constant-Z Load



(1) Bus k:

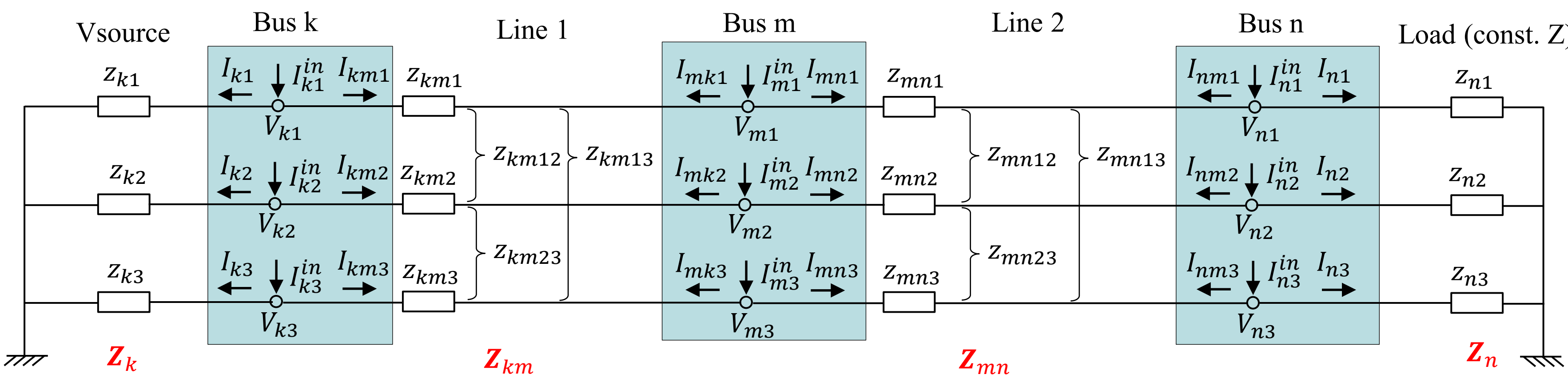
Injection currents at Bus k:

$$\begin{bmatrix} I_{k1}^{in} \\ I_{k2}^{in} \\ I_{k3}^{in} \end{bmatrix} = \begin{bmatrix} I_{k1} \\ I_{k2} \\ I_{k3} \end{bmatrix} + \begin{bmatrix} I_{km1} \\ I_{km2} \\ I_{km3} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{I}_k^{in} = \boldsymbol{I}_k + \boldsymbol{I}_{km} = \boldsymbol{Y}_k\boldsymbol{V}_k + \boldsymbol{Y}_{km}(\boldsymbol{V}_k - \boldsymbol{V}_m)$$
$$= (\boldsymbol{Y}_k + \boldsymbol{Y}_{km})\boldsymbol{V}_k - \boldsymbol{Y}_{km}\boldsymbol{V}_m$$

$$\text{where,} \boldsymbol{I}_k^{in} = \begin{bmatrix} I_{k1}^{in} \\ I_{k2}^{in} \\ I_{k3}^{in} \end{bmatrix}.$$

2 Example 1: Constant-Z Load



(2) Bus m:

1) Branch currents of Line 1

According to Kirchhoff's voltage law:

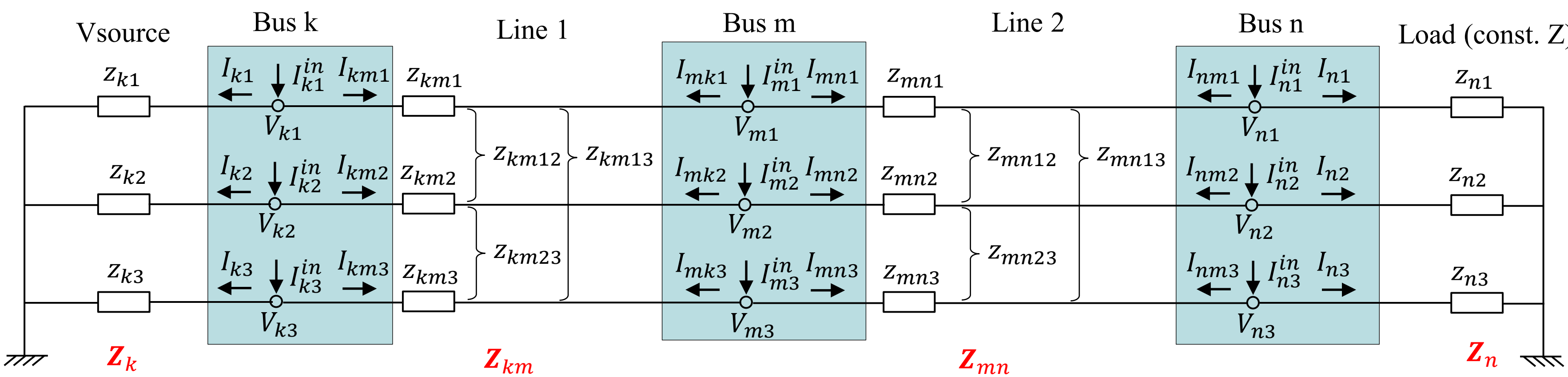
Strictly, z_{km*} should be z_{mk*}. For a line, $z_{km*} = z_{mk*}$, so z_{km*} is used here.

$$\begin{bmatrix} V_{m1} \\ V_{m2} \\ V_{m3} \end{bmatrix} = \begin{bmatrix} V_{k1} \\ V_{k2} \\ V_{k3} \end{bmatrix} + \begin{bmatrix} z_{km1} & z_{km12} & z_{km13} \\ z_{km12} & z_{km2} & z_{km23} \\ z_{km13} & z_{km23} & z_{km3} \end{bmatrix} \begin{bmatrix} I_{mk1} \\ I_{mk2} \\ I_{mk3} \end{bmatrix} \Rightarrow \begin{bmatrix} I_{mk1} \\ I_{mk2} \\ I_{mk3} \end{bmatrix} = \begin{bmatrix} z_{km1} & z_{km12} & z_{km13} \\ z_{km12} & z_{km2} & z_{km23} \\ z_{km13} & z_{km23} & z_{km3} \end{bmatrix}^{-1} \begin{bmatrix} V_{m1} - V_{k1} \\ V_{m2} - V_{k2} \\ V_{m3} - V_{k3} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} I_{mk1} \\ I_{mk2} \\ I_{mk3} \end{bmatrix} = \begin{bmatrix} y_{km1} & y_{km12} & y_{km13} \\ y_{km12} & y_{km2} & y_{km23} \\ y_{km13} & y_{km23} & y_{km3} \end{bmatrix} \begin{bmatrix} V_{m1} - V_{k1} \\ V_{m2} - V_{k2} \\ V_{m3} - V_{k3} \end{bmatrix} \Rightarrow \boldsymbol{I}_{mk} = \boldsymbol{Y}_{km}(\boldsymbol{V}_m - \boldsymbol{V}_k)$$

where, $\boldsymbol{I}_{mk} = \begin{bmatrix} I_{mk1} \\ I_{mk2} \\ I_{mk3} \end{bmatrix}$.

2 Example 1: Constant-Z Load



(2) Bus m:

2) Branch currents of Line 2

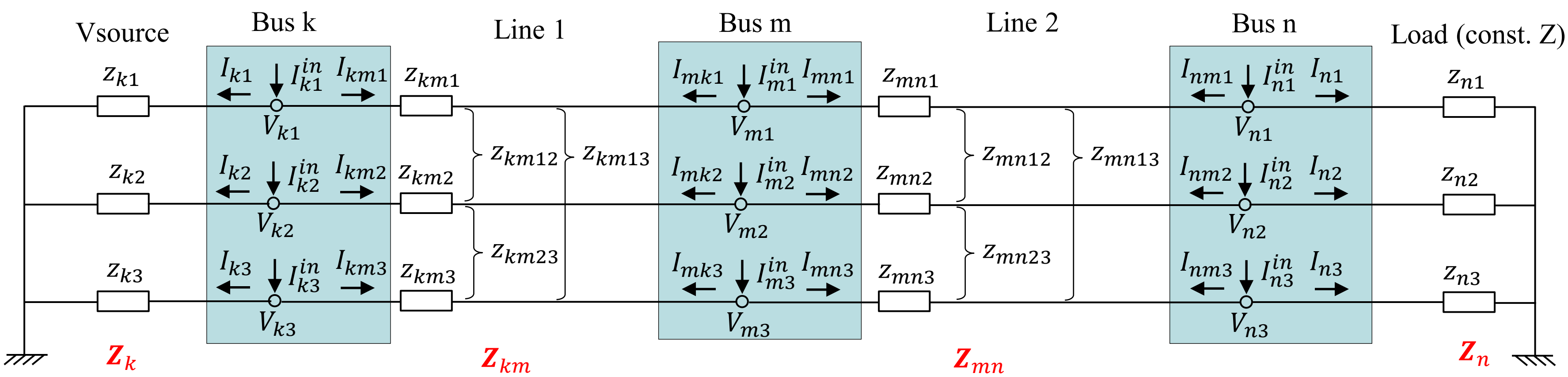
According to Kirchhoff's voltage law:

$$\begin{bmatrix} V_{m1} \\ V_{m2} \\ V_{m3} \end{bmatrix} = \begin{bmatrix} V_{n1} \\ V_{n2} \\ V_{n3} \end{bmatrix} + \begin{bmatrix} z_{mn1} & z_{mn12} & z_{mn13} \\ z_{mn12} & z_{mn2} & z_{mn23} \\ z_{mn13} & z_{mn23} & z_{mn3} \end{bmatrix} \begin{bmatrix} I_{mn1} \\ I_{mn2} \\ I_{mn3} \end{bmatrix} \Rightarrow \begin{bmatrix} I_{mn1} \\ I_{mn2} \\ I_{mn3} \end{bmatrix} = \begin{bmatrix} z_{mn1} & z_{mn12} & z_{mn13} \\ z_{mn12} & z_{mn2} & z_{mn23} \\ z_{mn13} & z_{mn23} & z_{mn3} \end{bmatrix}^{-1} \begin{bmatrix} V_{m1} - V_{n1} \\ V_{m2} - V_{n2} \\ V_{m3} - V_{n3} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} I_{mn1} \\ I_{mn2} \\ I_{mn3} \end{bmatrix} = \begin{bmatrix} y_{mn1} & y_{mn12} & y_{mn13} \\ y_{mn12} & y_{mn2} & y_{mn23} \\ y_{mn13} & y_{mn23} & y_{mn3} \end{bmatrix} \begin{bmatrix} V_{m1} - V_{n1} \\ V_{m2} - V_{n2} \\ V_{m3} - V_{n3} \end{bmatrix} \Rightarrow \boldsymbol{I}_{mn} = \boldsymbol{Y}_{mn}(\boldsymbol{V}_m - \boldsymbol{V}_n)$$

where, $\boldsymbol{I}_{mn} = \begin{bmatrix} I_{mn1} \\ I_{mn2} \\ I_{mn3} \end{bmatrix}, \boldsymbol{V}_n = \begin{bmatrix} V_{n1} \\ V_{n2} \\ V_{n3} \end{bmatrix}.$

2 Example 1: Constant-Z Load



(2) Bus m:

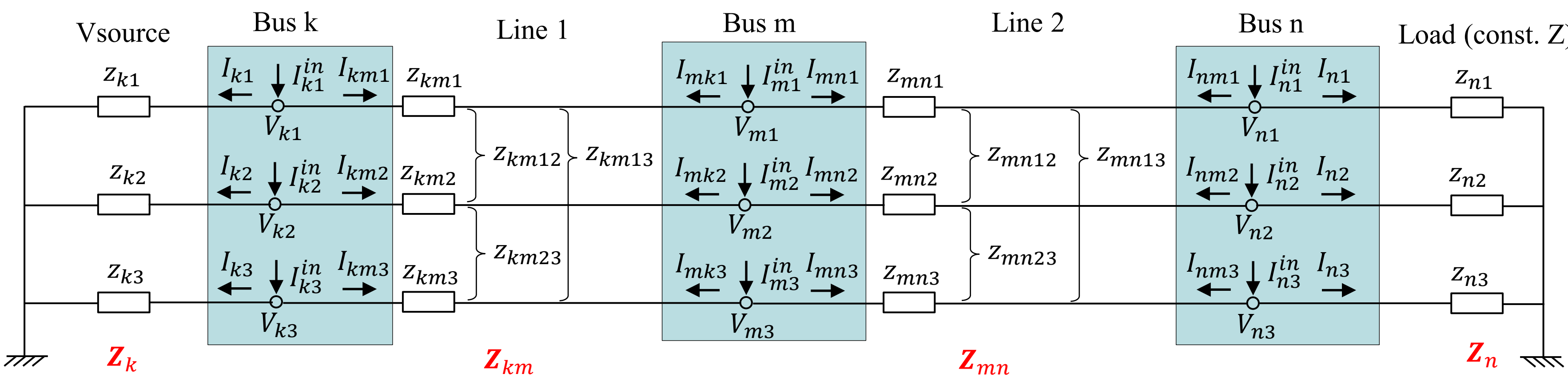
Injection currents at Bus m:

$$\begin{bmatrix} I_{m1}^{in} \\ I_{m2}^{in} \\ I_{m3}^{in} \end{bmatrix} = \begin{bmatrix} I_{mk1} \\ I_{mk2} \\ I_{mk3} \end{bmatrix} + \begin{bmatrix} I_{mn1} \\ I_{mn2} \\ I_{mn3} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{I}_m^{in} = \boldsymbol{I}_{mk} + \boldsymbol{I}_{mn} = \boldsymbol{Y}_{km}(\boldsymbol{V}_m - \boldsymbol{V}_k) + \boldsymbol{Y}_{mn}(\boldsymbol{V}_m - \boldsymbol{V}_n)$$
$$= (\boldsymbol{Y}_{km} + \boldsymbol{Y}_{mn})\boldsymbol{V}_m - \boldsymbol{Y}_{km}\boldsymbol{V}_k - \boldsymbol{Y}_{mn}\boldsymbol{V}_n$$

$$\text{where,} \boldsymbol{I}_m^{in} = \begin{bmatrix} I_{m1}^{in} \\ I_{m2}^{in} \\ I_{m3}^{in} \end{bmatrix}.$$

2 Example 1: Constant-Z Load



(3) Bus n:

1) Branch currents of Line 2

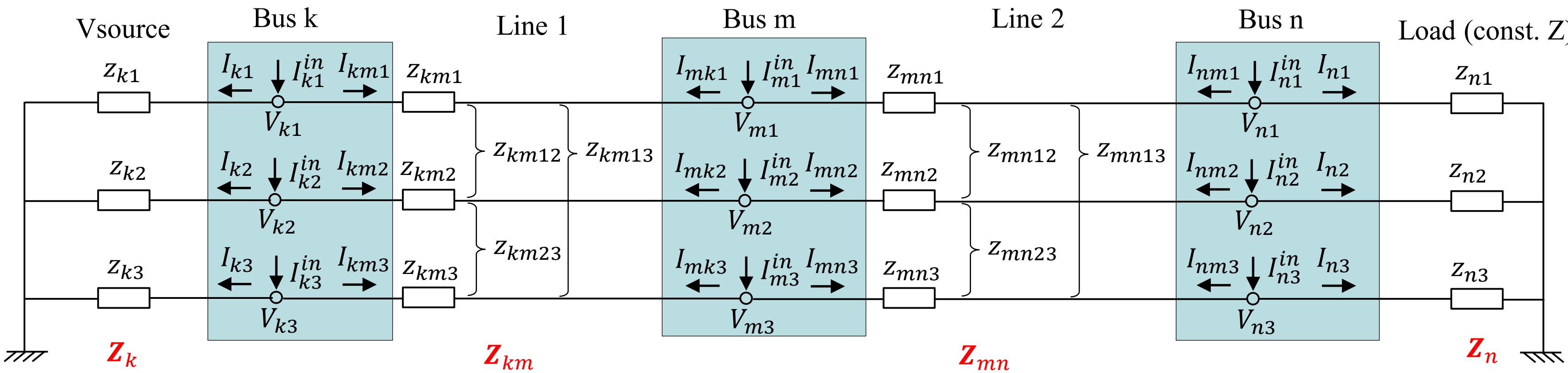
According to Kirchhoff's voltage law:

$$\begin{bmatrix} V_{n1} \\ V_{n2} \\ V_{n3} \end{bmatrix} = \begin{bmatrix} V_{m1} \\ V_{m2} \\ V_{m3} \end{bmatrix} + \begin{bmatrix} z_{mn1} & z_{mn12} & z_{mn13} \\ z_{mn12} & z_{mn2} & z_{mn23} \\ z_{mn13} & z_{mn23} & z_{mn3} \end{bmatrix} \begin{bmatrix} I_{nm1} \\ I_{nm2} \\ I_{nm3} \end{bmatrix} \Rightarrow \begin{bmatrix} I_{nm1} \\ I_{nm2} \\ I_{nm3} \end{bmatrix} = \begin{bmatrix} z_{mn1} & z_{mn12} & z_{mn13} \\ z_{mn12} & z_{mn2} & z_{mn23} \\ z_{mn13} & z_{mn23} & z_{mn3} \end{bmatrix}^{-1} \begin{bmatrix} V_{n1} - V_{m1} \\ V_{n2} - V_{m2} \\ V_{n3} - V_{m3} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} I_{nm1} \\ I_{nm2} \\ I_{nm3} \end{bmatrix} = \begin{bmatrix} y_{mn1} & y_{mn12} & y_{mn13} \\ y_{mn12} & y_{mn2} & y_{mn23} \\ y_{mn13} & y_{mn23} & y_{mn3} \end{bmatrix} \begin{bmatrix} V_{n1} - V_{m1} \\ V_{n2} - V_{m2} \\ V_{n3} - V_{m3} \end{bmatrix} \Rightarrow \boldsymbol{I}_{nm} = \boldsymbol{Y}_{mn}(\boldsymbol{V}_n - \boldsymbol{V}_m)$$

$$\text{where, } \boldsymbol{I}_{nm} = \begin{bmatrix} I_{nm1} \\ I_{nm2} \\ I_{nm3} \end{bmatrix}.$$

2 Example 1: Constant-Z Load



(3) Bus n:

2) Branch currents of Load

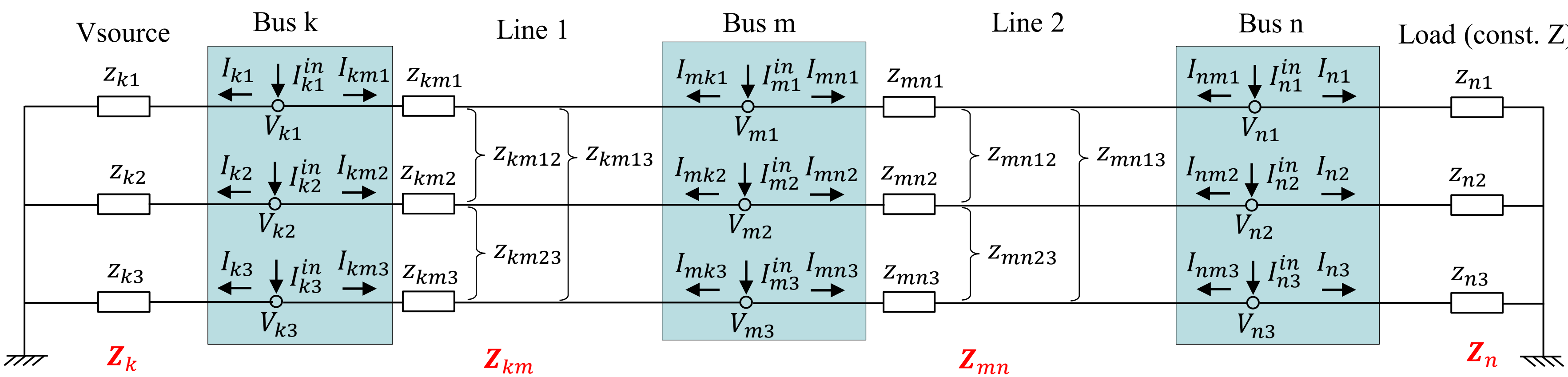
According to Kirchhoff's voltage law:

$$\begin{bmatrix} V_{n1} \\ V_{n2} \\ V_{n3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} z_{n1} & 0 & 0 \\ 0 & z_{n2} & 0 \\ 0 & 0 & z_{n3} \end{bmatrix} \begin{bmatrix} I_{n1} \\ I_{n2} \\ I_{n3} \end{bmatrix} \Rightarrow \begin{bmatrix} I_{n1} \\ I_{n2} \\ I_{m3} \end{bmatrix} = \begin{bmatrix} z_{n1} & 0 & 0 \\ 0 & z_{n2} & 0 \\ 0 & 0 & z_{n3} \end{bmatrix}^{-1} \begin{bmatrix} V_{n1} \\ V_{n2} \\ V_{n3} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} I_{n1} \\ I_{n2} \\ I_{m3} \end{bmatrix} = \begin{bmatrix} y_{n1} & 0 & 0 \\ 0 & y_{n2} & 0 \\ 0 & 0 & y_{n3} \end{bmatrix} \begin{bmatrix} V_{n1} \\ V_{n2} \\ V_{n3} \end{bmatrix} \Rightarrow \boldsymbol{I}_n = \boldsymbol{Y}_n \boldsymbol{V}_n$$

where, $\boldsymbol{I}_n = \begin{bmatrix} I_{n1} \\ I_{n2} \\ I_{m3} \end{bmatrix}$.

2 Example 1: Constant-Z Load



(3) Bus n:

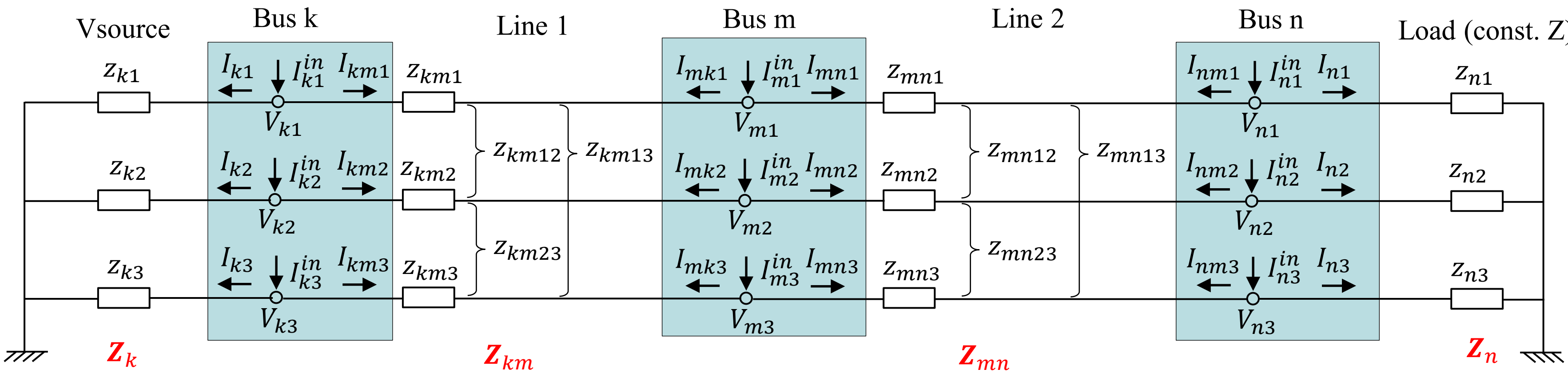
Injection currents at Bus n:

$$\begin{bmatrix} I_{n1}^{in} \\ I_{n2}^{in} \\ I_{n3}^{in} \end{bmatrix} = \begin{bmatrix} I_{nm1} \\ I_{nm2} \\ I_{nm3} \end{bmatrix} + \begin{bmatrix} I_{n1} \\ I_{n2} \\ I_{n3} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{I}_n^{in} = \boldsymbol{I}_{nm} + \boldsymbol{I}_n = \boldsymbol{Y}_{mn}(\boldsymbol{V}_n - \boldsymbol{V}_m) + \boldsymbol{Y}_n\boldsymbol{V}_n$$
$$= (\boldsymbol{Y}_{mn} + \boldsymbol{Y}_n)\boldsymbol{V}_n - \boldsymbol{Y}_{mn}\boldsymbol{V}_m$$

$$\text{where,} \boldsymbol{I}_n^{in} = \begin{bmatrix} I_{n1}^{in} \\ I_{n2}^{in} \\ I_{n3}^{in} \end{bmatrix}.$$

2 Example 1: Constant-Z Load



Put all the injection currents together:

$$\begin{cases} \boldsymbol{I}_k^{in} = (\boldsymbol{Y}_k + \boldsymbol{Y}_{km})\boldsymbol{V}_k - \boldsymbol{Y}_{km}\boldsymbol{V}_m \\ \boldsymbol{I}_m^{in} = (\boldsymbol{Y}_{km} + \boldsymbol{Y}_{mn})\boldsymbol{V}_m - \boldsymbol{Y}_{km}\boldsymbol{V}_k - \boldsymbol{Y}_{mn}\boldsymbol{V}_n \\ \boldsymbol{I}_n^{in} = (\boldsymbol{Y}_{mn} + \boldsymbol{Y}_n)\boldsymbol{V}_n - \boldsymbol{Y}_{mn}\boldsymbol{V}_m \end{cases}$$

$$\Rightarrow \begin{bmatrix} \boldsymbol{I}_k^{in} \\ \boldsymbol{I}_m^{in} \\ \boldsymbol{I}_n^{in} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_k + \boldsymbol{Y}_{km} & -\boldsymbol{Y}_{km} & \boldsymbol{0} \\ -\boldsymbol{Y}_{km} & \boldsymbol{Y}_{km} + \boldsymbol{Y}_{mn} & -\boldsymbol{Y}_{mn} \\ \boldsymbol{0} & -\boldsymbol{Y}_{mn} & \boldsymbol{Y}_{mn} + \boldsymbol{Y}_n \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_k \\ \boldsymbol{V}_m \\ \boldsymbol{V}_n \end{bmatrix}$$

$$\Rightarrow \mathbf{I}_{\text{inj}} = \mathbf{YV}$$

$$\text{where, } \mathbf{I}_{\text{inj}} = \begin{bmatrix} \boldsymbol{I}_k^{in} \\ \boldsymbol{I}_m^{in} \\ \boldsymbol{I}_n^{in} \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} \boldsymbol{Y}_k + \boldsymbol{Y}_{km} & -\boldsymbol{Y}_{km} & \boldsymbol{0} \\ -\boldsymbol{Y}_{km} & \boldsymbol{Y}_{km} + \boldsymbol{Y}_{mn} & -\boldsymbol{Y}_{mn} \\ \boldsymbol{0} & -\boldsymbol{Y}_{mn} & \boldsymbol{Y}_{mn} + \boldsymbol{Y}_n \end{bmatrix}, \mathbf{V} = \begin{bmatrix} \boldsymbol{V}_k \\ \boldsymbol{V}_m \\ \boldsymbol{V}_n \end{bmatrix}.$$

Conclusions for constructing $\mathbf{Y}$: (1) $\mathbf{Y}_{ij} = \mathbf{Y}_{ji}$; (2) $\mathbf{Y}_{ij} = -\boldsymbol{Y}_{ij}$; (3) $\mathbf{Y}_{ii} = \boldsymbol{Y}_i + \sum_{k=1,k\neq i}^{N} \boldsymbol{Y}_{ik}$

2 Example 1: Constant-Z Load

Exported primitive admittance matrix of each element from OpenDSS:

											G	B	
	Vsource												
$\boldsymbol{Y}_k$	0.2	0	0	0	0	0	-0.2	0	0	0	0	0	$-\boldsymbol{Y}_k$
	0	0	0.2	0	0	0	0	0	-0.2	0	0	0	
	0	0	0	0	0.2	0	0	0	0	0	-0.2	0	
$-\boldsymbol{Y}_k$	-0.2	0	0	0	0	0	0.2	0	0	0	0	0	$\boldsymbol{Y}_k$
	0	0	-0.2	0	0	0	0	0	0.2	0	0	0	
	0	0	0	0	-0.2	0	0	0	0	0	0.2	0	
	Line 1												
$\boldsymbol{Y}_{km}$	0.4314	-0.7738	-0.1235	0.1881	-0.1235	0.1881	-0.4314	0.7738	0.1235	-0.1881	0.1235	-0.1881	$-\boldsymbol{Y}_{km}$
	-0.1235	0.1881	0.4314	-0.7738	-0.1235	0.1881	0.1235	-0.1881	-0.4314	0.7738	0.1235	-0.1881	
	-0.1235	0.1881	-0.1235	0.1881	0.4314	-0.7738	0.1235	-0.1881	0.1235	-0.1881	-0.4314	0.7738	
$-\boldsymbol{Y}_{km}$	-0.4314	0.7738	0.1235	-0.1881	0.1235	-0.1881	0.4314	-0.7738	-0.1235	0.1881	-0.1235	0.1881	$\boldsymbol{Y}_{km}$
	0.1235	-0.1881	-0.4314	0.7738	0.1235	-0.1881	-0.1235	0.1881	0.4314	-0.7738	-0.1235	0.1881	
	0.1235	-0.1881	0.1235	-0.1881	-0.4314	0.7738	-0.1235	0.1881	-0.1235	0.1881	0.4314	-0.7738	
	Line 2												
$\boldsymbol{Y}_{mn}$	0.5176	-0.4304	-0.1491	0.0696	-0.1491	0.0696	-0.5176	0.4304	0.1491	-0.0696	0.1491	-0.0696	$-\boldsymbol{Y}_{mn}$
	-0.1491	0.0696	0.5176	-0.4304	-0.1491	0.0696	0.1491	-0.0696	-0.5176	0.4304	0.1491	-0.0696	
	-0.1491	0.0696	-0.1491	0.0696	0.5176	-0.4304	0.1491	-0.0696	0.1491	-0.0696	-0.5176	0.4304	
$-\boldsymbol{Y}_{mn}$	-0.5176	0.4304	0.1491	-0.0696	0.1491	-0.0696	0.5176	-0.4304	-0.1491	0.0696	-0.1491	0.0696	$\boldsymbol{Y}_{mn}$
	0.1491	-0.0696	-0.5176	0.4304	0.1491	-0.0696	-0.1491	0.0696	0.5176	-0.4304	-0.1491	0.0696	
	0.1491	-0.0696	0.1491	-0.0696	-0.5176	0.4304	-0.1491	0.0696	-0.1491	0.0696	0.5176	-0.4304	
	Load												
$\boldsymbol{Y}_n$	0.002625	0	0	0	0	0	-0.00263	0					
	0	0	0.002625	0	0	0	-0.00263	0					
	0	0	0	0	0.002625	0	-0.00263	0					
	-0.00263	0	-0.00263	0	-0.00263	0	0.007877	0					

Why the Vsource has 6 rows, while the Load has 4 rows? In OpenDSS, a 3-phase Vsource is a two-terminal object, and is defined as a voltage source behind an impedance, so it has 6 nodes. In contrast, a 3-phase wye-connected load only has one terminal, and only has 4 nodes, A, B, C and neutral.

2 Example 1: Constant-Z Load

Put all elements' Yprim together:

	Vsource												
	0.2	0	0	0	0	0	-0.2	0	0	0	0	0	
Y_k	0	0	0.2	0	0	0	0	0	-0.2	0	0	0	$-Y_k$
	0	0	0	0	0.2	0	0	0	0	0	-0.2	0	
	-0.2	0	0	0	0	0	0.2	0	0	0	0	0	
$-Y_k$	0	0	-0.2	0	0	0	0	0	0.2	0	0	0	Y_k
	0	0	0	0	-0.2	0	0	0	0	0	0.2	0	
	Line 1												
	0.4314	-0.7738	-0.1235	0.1881	-0.1235	0.1881	-0.4314	0.7738	0.1235	-0.1881	0.1235	-0.1881	
Y_{km}	-0.1235	0.1881	0.4314	-0.7738	-0.1235	0.1881	0.1235	-0.1881	-0.4314	0.7738	0.1235	-0.1881	$-Y_{km}$
	-0.1235	0.1881	-0.1235	0.1881	0.4314	-0.7738	0.1235	-0.1881	0.1235	-0.1881	-0.4314	0.7738	
	-0.4314	0.7738	0.1235	-0.1881	0.1235	-0.1881	0.4314	-0.7738	-0.1235	0.1881	-0.1235	0.1881	
$-Y_{km}$	0.1235	-0.1881	-0.4314	0.7738	0.1235	-0.1881	-0.1235	0.1881	0.4314	-0.7738	-0.1235	0.1881	Y_{km}
	0.1235	-0.1881	0.1235	-0.1881	-0.4314	0.7738	-0.1235	0.1881	-0.1235	0.1881	0.4314	-0.7738	
	Line 2												
	0.5176	-0.4304	-0.1491	0.0696	-0.1491	0.0696	-0.5176	0.4304	0.1491	-0.0696	0.1491	-0.0696	
Y_{mn}	-0.1491	0.0696	0.5176	-0.4304	-0.1491	0.0696	0.1491	-0.0696	-0.5176	0.4304	0.1491	-0.0696	$-Y_{mn}$
	-0.1491	0.0696	-0.1491	0.0696	0.5176	-0.4304	0.1491	-0.0696	0.1491	-0.0696	-0.5176	0.4304	
	-0.5176	0.4304	0.1491	-0.0696	0.1491	-0.0696	0.5176	-0.4304	-0.1491	0.0696	-0.1491	0.0696	
$-Y_{mn}$	0.1491	-0.0696	-0.5176	0.4304	0.1491	-0.0696	-0.1491	0.0696	0.5176	-0.4304	-0.1491	0.0696	Y_{mn}
	0.1491	-0.0696	0.1491	-0.0696	-0.5176	0.4304	-0.1491	0.0696	-0.1491	0.0696	0.5176	-0.4304	
	Load												
	0.002625	0	0	0	0	0	-0.00263	0					
Y_n	0	0	0.002625	0	0	0	-0.00263	0					
	0	0	0	0	0.002625	0	-0.00263	0					
	-0.00263	0	-0.00263	0	-0.00263	0	0.007877	0					

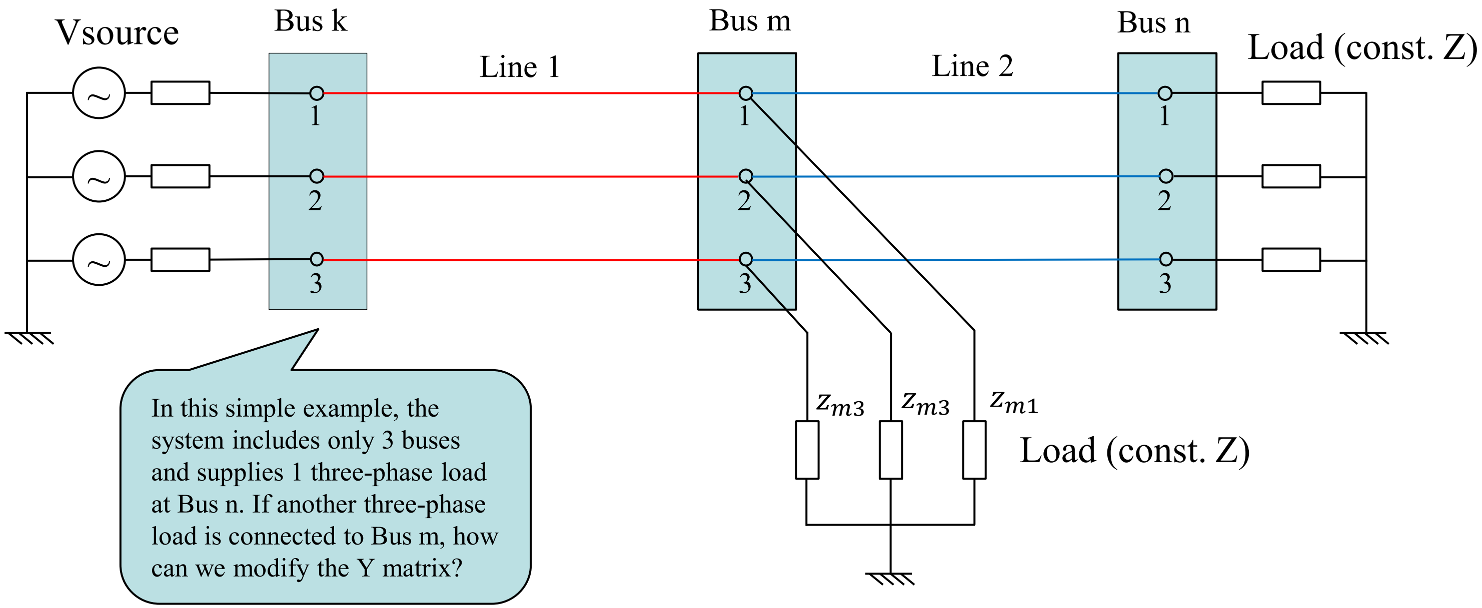
Constructing **Y**

$Y_k + Y_{km}$ $-Y_{km}$ $Y_{km} + Y_{mn}$ $-Y_{mn}$

	K.1		K.2		K.3		M.1		M.2		M.3		N.1		N.2		N.3	
K.1	0.6314	-0.7738	-0.1235	0.1881	-0.1235	0.1881	-0.4314	0.7738	0.1235	-0.1881	0.1235	-0.1881	0	0	0	0	0	0
K.2	-0.1235	0.1881	0.6314	-0.7738	-0.1235	0.1881	0.1235	-0.1881	-0.4314	0.7738	0.1235	-0.1881	0	0	0	0	0	0
K.3	-0.1235	0.1881	-0.1235	0.1881	0.6314	-0.7738	0.1235	-0.1881	0.1235	-0.1881	-0.4314	0.7738	0	0	0	0	0	0
M.1	-0.4314	0.7738	0.1235	-0.1881	0.1235	-0.1881	0.9490	-1.2042	-0.2726	0.2577	-0.2726	0.2577	-0.5176	0.4304	0.1491	-0.0696	0.1491	-0.0696
M.2	0.1235	-0.1881	-0.4314	0.7738	0.1235	-0.1881	-0.2726	0.2577	0.9490	-1.2042	-0.2726	0.2577	0.1491	-0.0696	-0.5176	0.4304	0.1491	-0.0696
M.3	0.1235	-0.1881	0.1235	-0.1881	-0.4314	0.7738	-0.2726	0.2577	-0.2726	0.2577	0.9490	-1.2042	0.1491	-0.0696	0.1491	-0.0696	-0.5176	0.4304
N.1	0	0	0	0	0	0	-0.5176	0.4304	0.1491	-0.0696	0.1491	-0.0696	0.5202	-0.4304	-0.1491	0.0696	-0.1491	0.0696
N.2	0	0	0	0	0	0	0.1491	-0.0696	-0.5176	0.4304	0.1491	-0.0696	-0.1491	0.0696	0.5202	-0.4304	-0.1491	0.0696
N.3	0	0	0	0	0	0	0.1491	-0.0696	0.1491	-0.0696	-0.5176	0.4304	-0.1491	0.0696	-0.1491	0.0696	0.5202	-0.4304

$-Y_{mn}$ $Y_n + Y_{mn}$

2 Example 1: Constant-Z Load

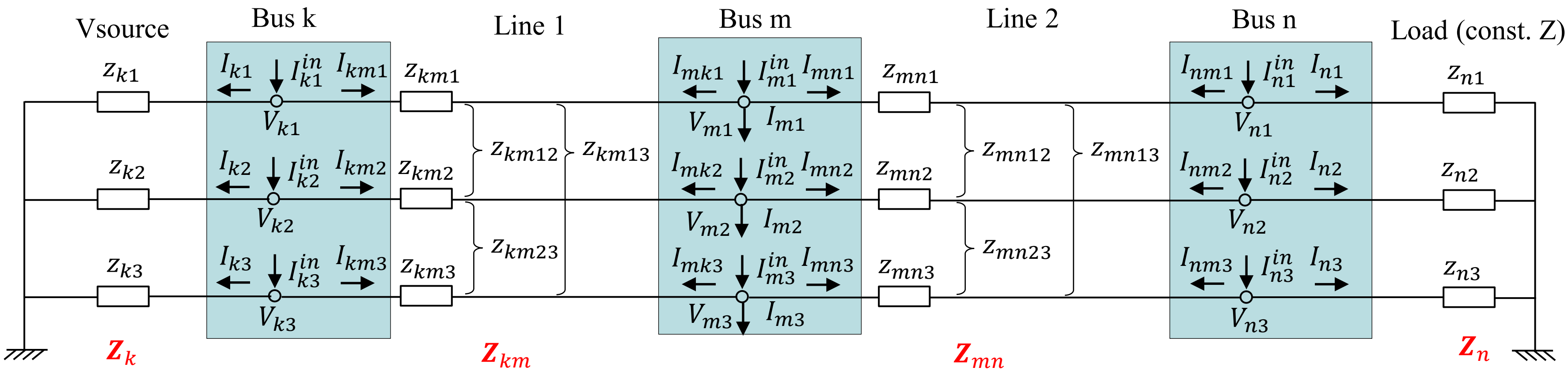


As shown in the figure, another constant-Z load is connected to Bus m. Then we can define:

$$\boldsymbol{Z}_m = \begin{bmatrix} z_{m1} & 0 & 0 \\ 0 & z_{m2} & 0 \\ 0 & 0 & z_{m3} \end{bmatrix}, \ \boldsymbol{Y}_m = \boldsymbol{Z}_m^{-1} = \begin{bmatrix} y_{m1} & 0 & 0 \\ 0 & y_{m2} & 0 \\ 0 & 0 & y_{m3} \end{bmatrix}$$

Then corresponding modifications is required for Bus m, while analysis for Bus k and Bus n remain unchanged.

2 Example 1: Constant-Z Load



(2) Bus m:

1) Branch currents of Line 1

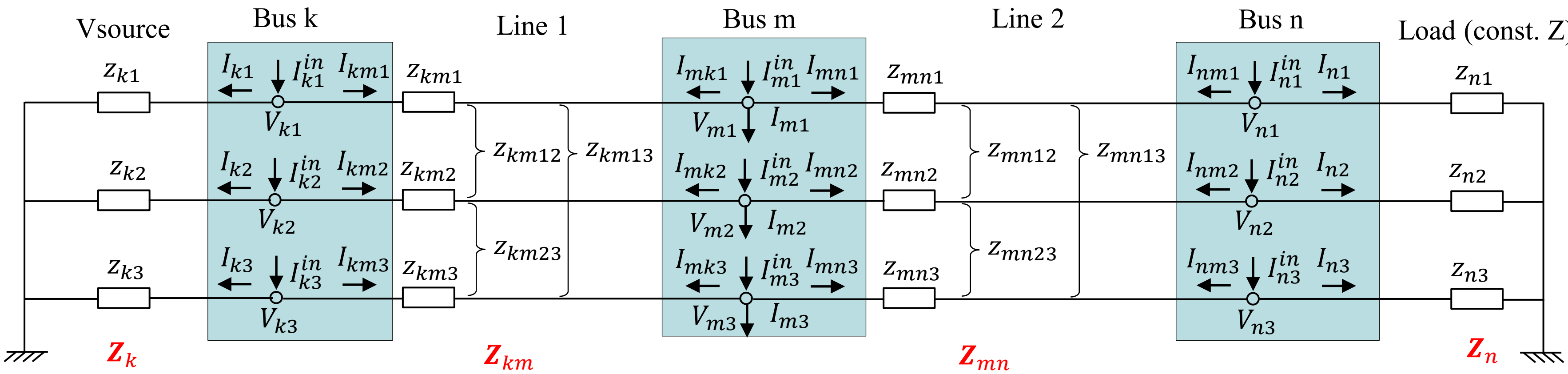
According to Kirchhoff's voltage law:

$$\begin{bmatrix} V_{m1} \\ V_{m2} \\ V_{m3} \end{bmatrix} = \begin{bmatrix} V_{k1} \\ V_{k2} \\ V_{k3} \end{bmatrix} + \begin{bmatrix} z_{km1} & z_{km12} & z_{km13} \\ z_{km12} & z_{km2} & z_{km23} \\ z_{km13} & z_{km23} & z_{km3} \end{bmatrix} \begin{bmatrix} I_{mk1} \\ I_{mk2} \\ I_{mk3} \end{bmatrix} \Rightarrow \begin{bmatrix} I_{mk1} \\ I_{mk2} \\ I_{mk3} \end{bmatrix} = \begin{bmatrix} z_{km1} & z_{km12} & z_{km13} \\ z_{km12} & z_{km2} & z_{km23} \\ z_{km13} & z_{km23} & z_{km3} \end{bmatrix}^{-1} \begin{bmatrix} V_{m1} - V_{k1} \\ V_{m2} - V_{k2} \\ V_{m3} - V_{k3} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} I_{mk1} \\ I_{mk2} \\ I_{mk3} \end{bmatrix} = \begin{bmatrix} y_{km1} & y_{km12} & y_{km13} \\ y_{km12} & y_{km2} & y_{km23} \\ y_{km13} & y_{km23} & y_{km3} \end{bmatrix} \begin{bmatrix} V_{m1} - V_{k1} \\ V_{m2} - V_{k2} \\ V_{m3} - V_{k3} \end{bmatrix} \Rightarrow \boldsymbol{I}_{mk} = \boldsymbol{Y}_{km}(\boldsymbol{V}_m - \boldsymbol{V}_k)$$

$$\text{where, } \boldsymbol{I}_{mk} = \begin{bmatrix} I_{mk1} \\ I_{mk2} \\ I_{mk3} \end{bmatrix}.$$

2 Example 1: Constant-Z Load



(2) Bus m:

2) Branch currents of Line 2

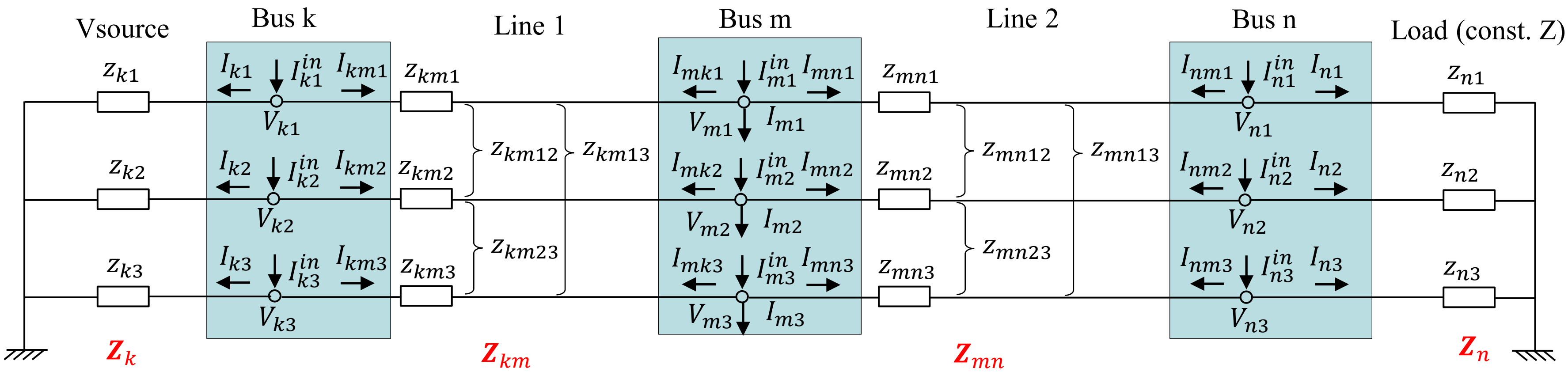
According to Kirchhoff's voltage law:

$$\begin{bmatrix} V_{m1} \\ V_{m2} \\ V_{m3} \end{bmatrix} = \begin{bmatrix} V_{n1} \\ V_{n2} \\ V_{n3} \end{bmatrix} + \begin{bmatrix} z_{mn1} & z_{mn12} & z_{mn13} \\ z_{mn12} & z_{mn2} & z_{mn23} \\ z_{mn13} & z_{mn23} & z_{mn3} \end{bmatrix} \begin{bmatrix} I_{mn1} \\ I_{mn2} \\ I_{mn3} \end{bmatrix} \Rightarrow \begin{bmatrix} I_{mn1} \\ I_{mn2} \\ I_{mn3} \end{bmatrix} = \begin{bmatrix} z_{mn1} & z_{mn12} & z_{mn13} \\ z_{mn12} & z_{mn2} & z_{mn23} \\ z_{mn13} & z_{mn23} & z_{mn3} \end{bmatrix}^{-1} \begin{bmatrix} V_{m1} - V_{n1} \\ V_{m2} - V_{n2} \\ V_{m3} - V_{n3} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} I_{mn1} \\ I_{mn2} \\ I_{mn3} \end{bmatrix} = \begin{bmatrix} y_{mn1} & y_{mn12} & y_{mn13} \\ y_{mn12} & y_{mn2} & y_{mn23} \\ y_{mn13} & y_{mn23} & y_{mn3} \end{bmatrix} \begin{bmatrix} V_{m1} - V_{n1} \\ V_{m2} - V_{n2} \\ V_{m3} - V_{n3} \end{bmatrix} \Rightarrow \boldsymbol{I}_{mn} = \boldsymbol{Y}_{mn}(\boldsymbol{V}_m - \boldsymbol{V}_n)$$

where, $\boldsymbol{I}_{mn} = \begin{bmatrix} I_{mn1} \\ I_{mn2} \\ I_{mn3} \end{bmatrix}, \boldsymbol{V}_n = \begin{bmatrix} V_{n1} \\ V_{n2} \\ V_{n3} \end{bmatrix}.$

2 Example 1: Constant-Z Load



(3) Bus m: (Note that the Load at Bus m is not drawn in the Figure due to limited space, but the load current $\boldsymbol{I}_m$ is marked.

2) Branch currents of Load

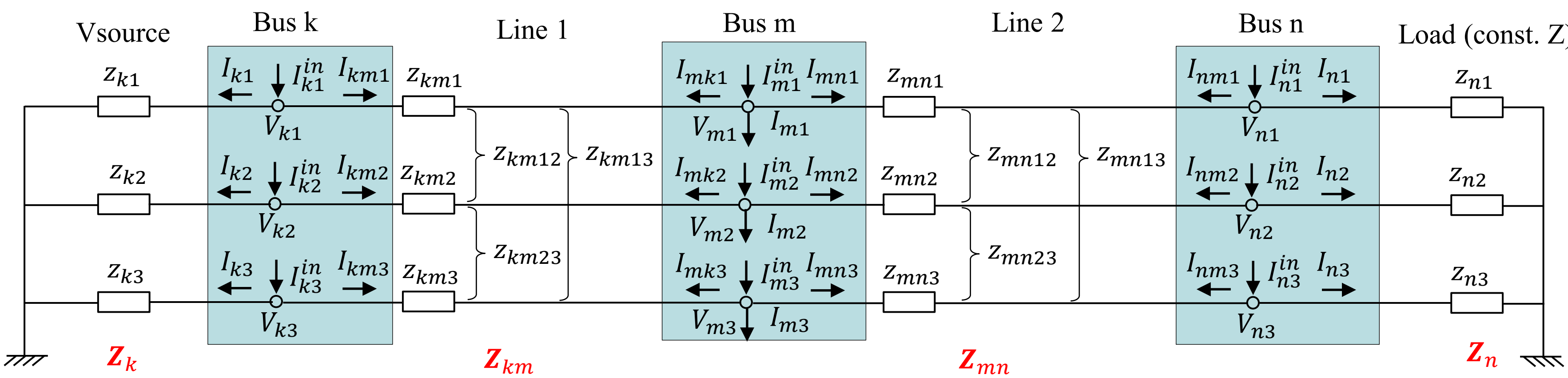
According to Kirchhoff's voltage law:

$$\begin{bmatrix} V_{m1} \\ V_{m2} \\ V_{m3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} z_{m1} & 0 & 0 \\ 0 & z_{m2} & 0 \\ 0 & 0 & z_{m3} \end{bmatrix} \begin{bmatrix} I_{m1} \\ I_{m2} \\ I_{m3} \end{bmatrix} \Rightarrow \begin{bmatrix} I_{m1} \\ I_{m2} \\ I_{m3} \end{bmatrix} = \begin{bmatrix} z_{m1} & 0 & 0 \\ 0 & z_{m2} & 0 \\ 0 & 0 & z_{m3} \end{bmatrix}^{-1} \begin{bmatrix} V_{m1} \\ V_{m2} \\ V_{m3} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} I_{m1} \\ I_{m2} \\ I_{m3} \end{bmatrix} = \begin{bmatrix} y_{m1} & 0 & 0 \\ 0 & y_{m2} & 0 \\ 0 & 0 & y_{m3} \end{bmatrix} \begin{bmatrix} V_{m1} \\ V_{m2} \\ V_{m3} \end{bmatrix} \Rightarrow \boldsymbol{I}_m = \boldsymbol{Y}_m \boldsymbol{V}_m$$

where, $\boldsymbol{I}_m = \begin{bmatrix} I_{m1} \\ I_{m2} \\ I_{m3} \end{bmatrix}$.

2 Example 1: Constant-Z Load



(2) Bus m:

Injection currents at Bus m:

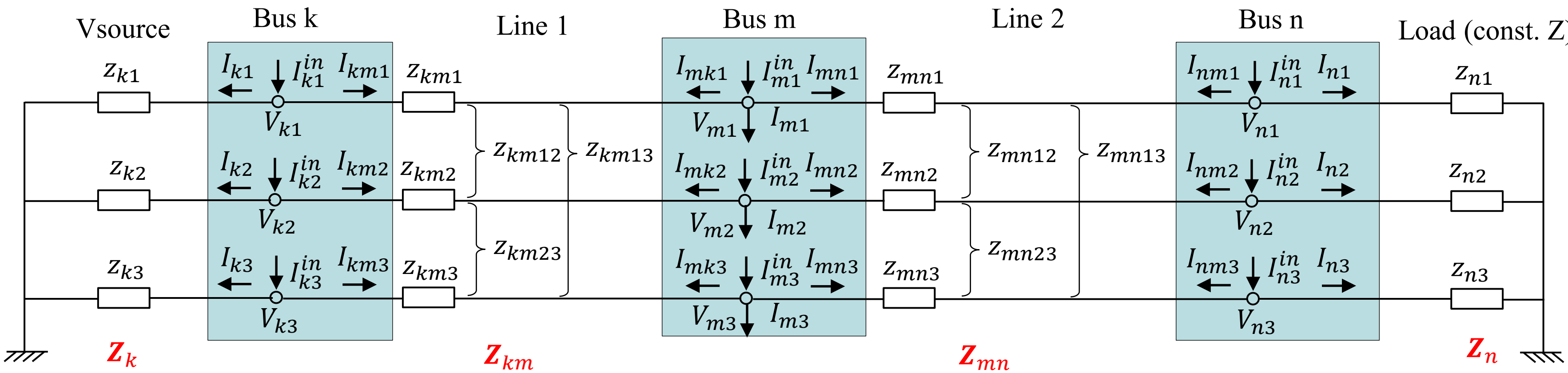
$$\begin{bmatrix} I_{m1}^{in} \\ I_{m2}^{in} \\ I_{m3}^{in} \end{bmatrix} = \begin{bmatrix} I_{mk1} \\ I_{mk2} \\ I_{mk3} \end{bmatrix} + \begin{bmatrix} I_{mn1} \\ I_{mn2} \\ I_{mn3} \end{bmatrix} + \begin{bmatrix} I_{m1} \\ I_{m2} \\ I_{m3} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{I}_m^{in} = \boldsymbol{I}_{mk} + \boldsymbol{I}_{mn} + \boldsymbol{I}_m = \boldsymbol{Y}_{km}(\boldsymbol{V}_m - \boldsymbol{V}_k) + \boldsymbol{Y}_{mn}(\boldsymbol{V}_m - \boldsymbol{V}_n) + \boldsymbol{Y}_m \boldsymbol{V}_m$$

$$= (\boldsymbol{Y}_{km} + \boldsymbol{Y}_{mn} + \boldsymbol{Y}_m)\boldsymbol{V}_m - \boldsymbol{Y}_{km}\boldsymbol{V}_k - \boldsymbol{Y}_{mn}\boldsymbol{V}_n$$

$$\text{where,} \boldsymbol{I}_m^{in} = \begin{bmatrix} I_{m1}^{in} \\ I_{m2}^{in} \\ I_{m3}^{in} \end{bmatrix}.$$

2 Example 1: Constant-Z Load



Finally, put all the injection currents together:

$$\begin{cases} I_k^{in} = (Y_k + Y_{km})V_k - Y_{km}V_m \\ I_m^{in} = (Y_{km} + Y_{mn} + Y_m)V_m - Y_{km}V_k - Y_{mn}V_n \\ I_n^{in} = (Y_{mn} + Y_n)V_n - Y_{mn}V_m \end{cases}$$

$$\Rightarrow \begin{bmatrix} I_k^{in} \\ I_m^{in} \\ I_n^{in} \end{bmatrix} = \begin{bmatrix} Y_k + Y_{km} & -Y_{km} & \mathbf{0} \\ -Y_{km} & Y_m + Y_{km} + Y_{mn} & -Y_{mn} \\ \mathbf{0} & -Y_{mn} & Y_{mn} + Y_n \end{bmatrix} \begin{bmatrix} V_k \\ V_m \\ V_n \end{bmatrix}$$

$$\Rightarrow \mathbf{I}_{\text{inj}} = \mathbf{YV}$$

where, $\mathbf{I}_{\text{inj}} = \begin{bmatrix} I_k^{in} \\ I_m^{in} \\ I_n^{in} \end{bmatrix}$, $\mathbf{Y} = \begin{bmatrix} Y_k + Y_{km} & -Y_{km} & \mathbf{0} \\ -Y_{km} & Y_m + Y_{km} + Y_{mn} & -Y_{mn} \\ \mathbf{0} & -Y_{mn} & Y_{mn} + Y_n \end{bmatrix}$, $\mathbf{V} = \begin{bmatrix} V_k \\ V_m \\ V_n \end{bmatrix}$.

Conclusions for constructing $\mathbf{Y}$:

(1) $\mathbf{Y}_{ij} = \mathbf{Y}_{ji}$, and $\mathbf{Y}_{ij} = -Y_{ij}$. It means the off-diagonal sub-matrices correspond to the negative values of the Y matrices of the power delivery elements connecting bus *i* and *j*.

(2) $\mathbf{Y}_{ii} = Y_i + \sum_{k=1,k\neq i}^{N} Y_{ik}$. The diagonal sub-matrices are the sum of Y matrices of all the elements connected to bus *i*.

2 Example 1: Constant-Z Load

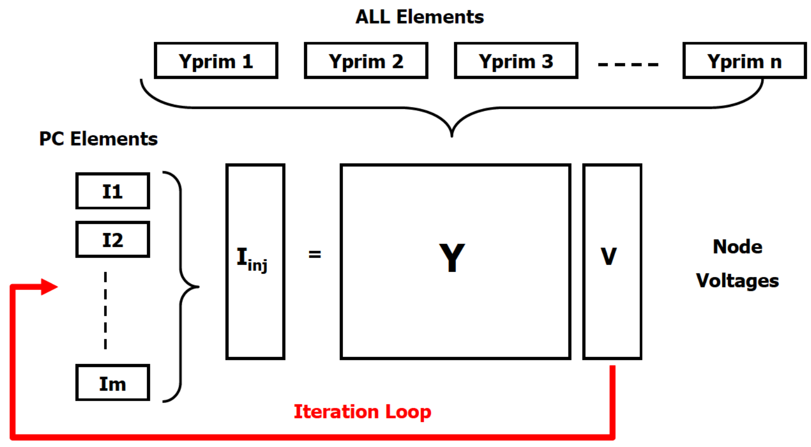


Figure 17. OpenDSS Solution Loop

A general outline of solution of the power flow:

- Step 1: Initialization.
  - Calculate the injection currents of the Vsource: $\boldsymbol{I}_{source} = \boldsymbol{Y}_{source} * \boldsymbol{V}_{source}$
  - Calculate the initial nodal voltages using $\boldsymbol{I}_{source}$ with all other injections currents set to 0: $\mathbf{V}_0 = \mathbf{Y}^{-1} * \mathbf{I}_{\text{inj},0}$
- Step 2:
  Calculate the injection (compensation) currents for all the power conversion (PC) elements and adding them into the appropriate slot in the current vector to obtain $\mathbf{I}_{inj,n}$, where, n denotes the n'th iteration.
- Step 3:
  Update voltage vector: $\mathbf{V}_{n+1} = \mathbf{Y}^{-1} * \mathbf{I}_{\text{inj},n}$
- Step 4:
  Check whether the voltage error exceeds the tolerance. If not, go to Step 2.

Note, regarding the injection currents:
1. $\boldsymbol{I}_{source}$ does not change in each iteration;
2. The currents for a non-constant-Z injection load will change in each iteration.
3. The currents for a constant-Z load will be zero in each iteration, since there's no compensate current.

2 Example 1: Constant-Z Load

$\mathbf{Y}^{-1}=$

																R	X
4.9355	0.0004	0.0001	0.0002	0.0001	0.0002	4.9274	-0.0152	-0.0022	-0.0053	-0.0022	-0.0053	4.9119	-0.0307	-0.0052	-0.0116	-0.0052	-0.0116
0.0001	0.0002	4.9355	0.0004	0.0001	0.0002	-0.0022	-0.0053	4.9274	-0.0152	-0.0022	-0.0053	-0.0052	-0.0116	4.9119	-0.0307	-0.0052	-0.0116
0.0001	0.0002	0.0001	0.0002	4.9355	0.0004	-0.0022	-0.0053	-0.0022	-0.0053	4.9274	-0.0152	-0.0052	-0.0116	-0.0052	-0.0116	4.9119	-0.0307
4.9274	-0.0152	-0.0022	-0.0053	-0.0022	-0.0053	5.5428	1.1746	0.168	0.4164	0.168	0.4164	5.5301	1.1525	0.1674	0.4063	0.1674	0.4063
-0.0022	-0.0053	4.9274	-0.0152	-0.0022	-0.0053	0.168	0.4164	5.5428	1.1746	0.168	0.4164	0.1674	0.4063	5.5301	1.1525	0.1674	0.4063
-0.0022	-0.0053	-0.0022	-0.0053	4.9274	-0.0152	0.168	0.4164	0.168	0.4164	5.5428	1.1746	0.1674	0.4063	0.1674	0.4063	5.5301	1.1525
4.9119	-0.0307	-0.0052	-0.0116	-0.0052	-0.0116	5.5301	1.1525	0.1674	0.4063	0.1674	0.4063	6.7083	2.3318	0.3988	0.8812	0.3988	0.8812
-0.0052	-0.0116	4.9119	-0.0307	-0.0052	-0.0116	0.1674	0.4063	5.5301	1.1525	0.1674	0.4063	0.3988	0.8812	6.7083	2.3318	0.3988	0.8812
-0.0052	-0.0116	-0.0052	-0.0116	4.9119	-0.0307	0.1674	0.4063	0.1674	0.4063	5.5301	1.1525	0.3988	0.8812	0.3988	0.8812	6.7083	2.3318

$$\boldsymbol{I}_{source} = \begin{bmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \end{bmatrix} = \boldsymbol{Y}_{source} * \boldsymbol{V}_{source}$$
$$= \begin{bmatrix} 1625.36 + j0 \\ -812.68 - j1407.6 \\ -812.68 + j1407.6 \end{bmatrix} A$$

$$\mathbf{I}_{\text{inj},0} = \begin{bmatrix} I_{k1,0}^{in} \\ I_{k2,0}^{in} \\ I_{k3,0}^{in} \\ I_{m1,0}^{in} \\ I_{m2,0}^{in} \\ I_{m3,0}^{in} \\ I_{n1,0}^{in} \\ I_{n2,0}^{in} \\ I_{n3,0}^{in} \end{bmatrix} = \begin{bmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1625.36 + j0 \\ -812.68 - j1407.6 \\ -812.68 + j1407.6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

2 Example 1: Constant-Z Load

Matlab code for solving power flow:

```
clear;

j = sqrt(-1);

%% Compute the Y matrix of each element
% Vsource
Z_k = [5+j*0, 0, 0; 0, 5+j*0, 0; 0, 0, 5+j*0];
Y_k = inv(Z_k);

% line 1
Rmatrix_1 = [0.62, 0.17, 0.17; 0.17, 0.62, 0.17; 0.17, 0.17, 0.62];
Xmatrix_1 = [1.21, 0.43, 0.43; 0.43, 1.21, 0.43; 0.43, 0.43, 1.21];

Z_km = Rmatrix_1 * 1 + j * Xmatrix_1 * 1;
Y_km = inv(Z_km);

% line 2
Rmatrix_2 = [1.19, 0.23, 0.23; 0.23, 1.19, 0.23; 0.23, 0.23, 1.19];
Xmatrix_2 = [1.21, 0.49, 0.49; 0.49, 1.21, 0.49; 0.49, 0.49, 1.21];
Z_mn = Rmatrix_2 * 1 + j * Xmatrix_2 * 1;
Y_mn = inv(Z_mn);

% load
S_load = 500 + j * 0;
S_n = [S_load/3; S_load/3; S_load/3];
V_load_mag = 13.8;
Z_LN = 1000*V_load_mag^2/conj(S_load);
Z_n = [Z_LN, 0, 0; 0, Z_LN, 0; 0, 0, Z_LN];
Y_n = inv(Z_n);

% system Y matrix
Y = [Y_k + Y_km, -Y_km, zeros(3,3); ...
     -Y_km, Y_km + Y_mn, -Y_mn; ...
     zeros(3,3), -Y_mn, Y_mn + Y_n];

%% initialization
I_source = [(1.02*13800/3^0.5)/5; (1.02*13800/3^0.5)/5 * (-1/2-j*3^0.5/2); (1.02*13800/3^0.5)/5 * (-1/2+j*3^0.5/2)];
I_0 = zeros(9, 1);
I_0(1:3, :) = I_source;

V_0 = Y\I_0;  % for a constant impedance load, we do not have to perform iterations
```

A constant-Z load does not need a compensation current. This is the final voltage vector.

2 Example 1: Constant-Z Load

Result Comparison:

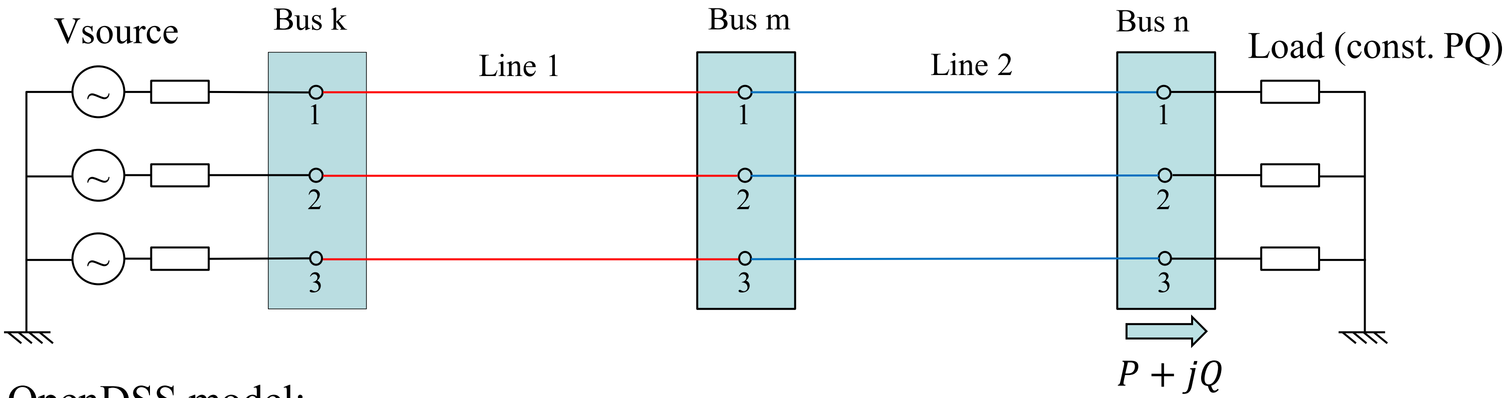
Node	Real	Imaginary
Vk1	8021.87	0.41
Vk2	-4010.58	-6947.34
Vk3	-4011.28	6946.94
Vm1	8012.36	-15.92
Vm2	-4019.97	-6930.94
Vm3	-3992.39	6946.87
Vn1	7992.16	-30.95
Vn2	-4022.89	-6905.93
Vn3	-3969.27	6936.89

Computed voltages (volts) using Matlab

Node	Real	Imaginary
Vk1	8021.87	0.00
Vk2	-4010.94	-6947.14
Vk3	-4010.94	6947.14
Vm1	8012.38	-15.38
Vm2	-4019.51	-6931.23
Vm3	-3992.87	6946.61
Vn1	7992.17	-30.69
Vn2	-4022.66	-6906.08
Vn3	-3969.51	6936.77

Exported voltages (volts) from OpenDSS

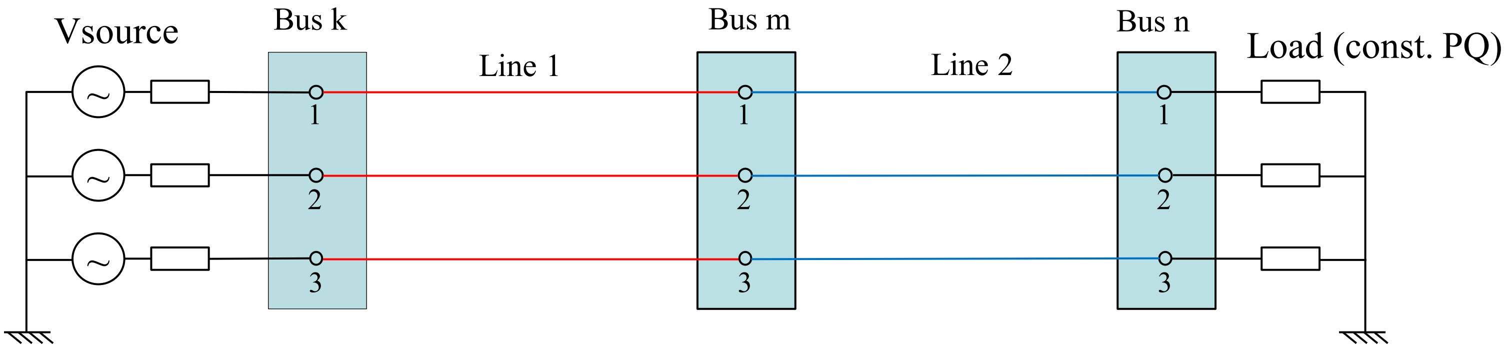
3 Example 2: Constant-PQ Load



OpenDSS model:

```
Clear
//-------------- Define a new circuit --------------------//
new circuit.experiment

//---------------Define a Voltage source------------------//
Edit "Vsource.source" phases=3 basekv=13.8 Pu=1.02 bus1=K R1=5 X1=0 R0=5 X0=0

//----------------- Define line codes  --------------------//
New LineCode.Code1 nphases= 3 Units= mi
 ~ Rmatrix= (0.62 | 0.17 0.62 | 0.17 0.17 0.62 )
 ~ Xmatrix= (1.21 | 0.43 1.21 | 0.43 0.43 1.21 )

New LineCode.Code2 nphases= 3 Units= mi
 ~ Rmatrix= (1.19 | 0.23 1.19 | 0.23 0.23 1.19 )
 ~ Xmatrix= (1.21 | 0.49 1.21 | 0.49 0.49 1.21 )
… (continued)
```

3 Example 2: Constant-PQ Load

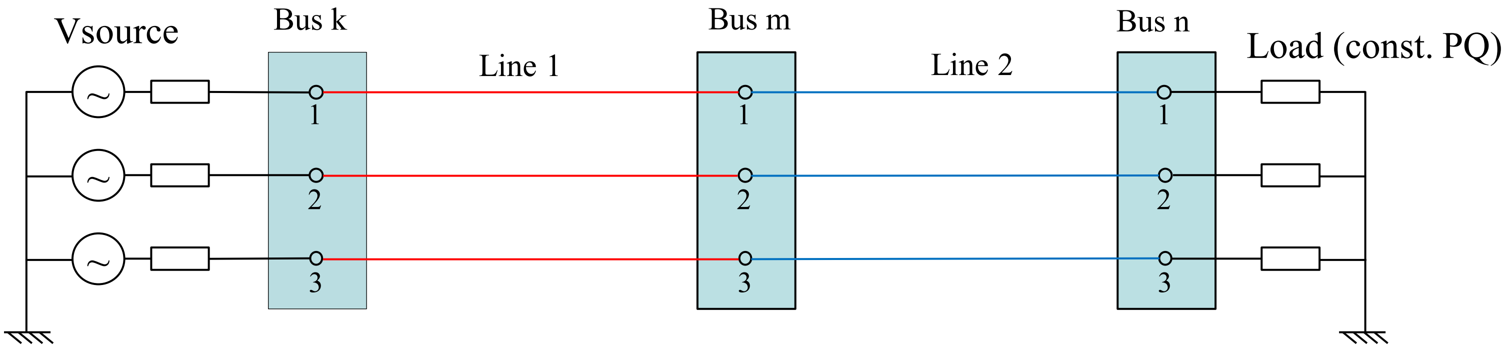


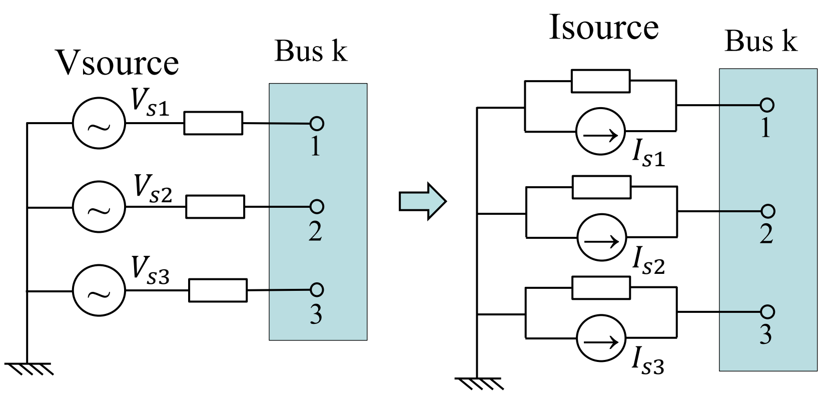
OpenDSS model:

```
… (continued)
//---------------  Define line 1 and 2  ------------------//
New Line.1 phases=3  Bus1=k.1.2.3 Bus2=m.1.2.3  length=1 units=mi LineCode=Code1
New Line.2 phases=3  Bus1=m.1.2.3 Bus2=n.1.2.3  length=1 units=mi LineCode=Code2

//----------------  Define a load   -----------------------//
New Load.L Bus1=n.1.2.3.0       Phases=3 Conn=wye Model=1 kV=13.8   kW=500 kvar=500

Set VoltageBases = "13.8"
Solve
```

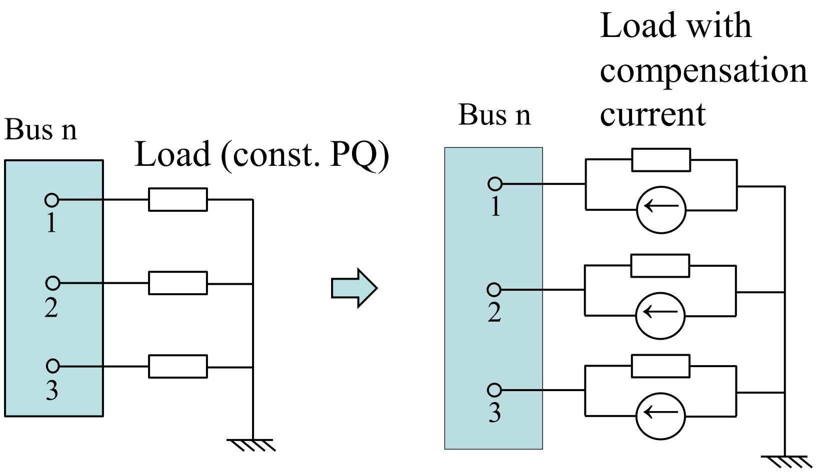
Constant-PQ load

3 Example 2: Constant-PQ Load





Thevinen equivalent to Norton equivalent



Various loads to a generic load model
(Constant Z + compensation current)

2 Example 1: Constant-Z Load

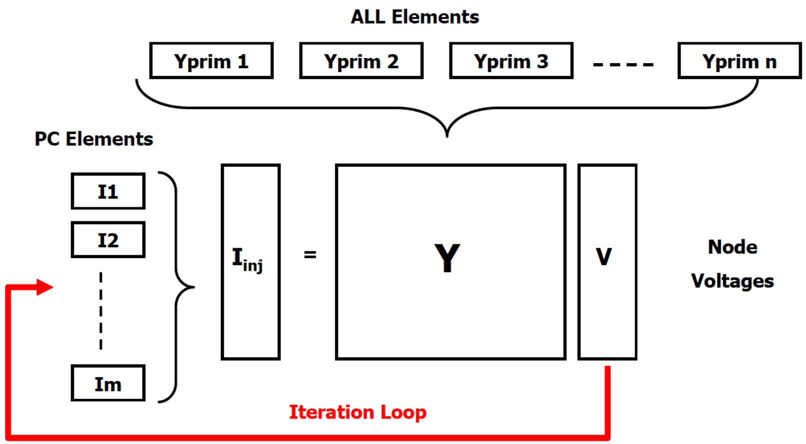


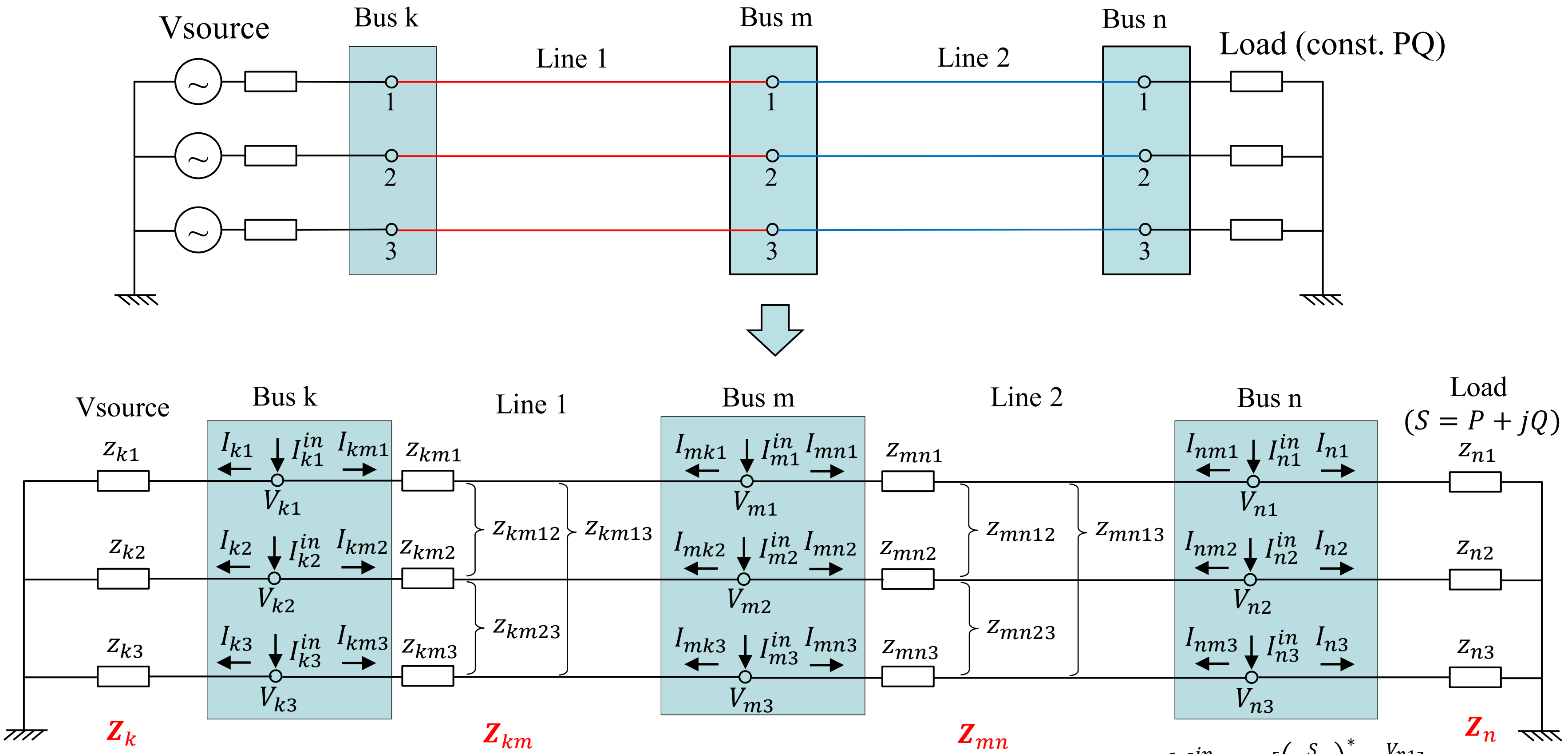
Figure 17. OpenDSS Solution Loop

A general outline of solution of the power flow:

- Step 1: Initialization.
  - Calculate the injection currents of the Vsource: $\boldsymbol{I}_{source} = \boldsymbol{Y}_{source} * \boldsymbol{V}_{source}$
  - Calculate the initial nodal voltages using $\boldsymbol{I}_{source}$ with all other injections currents set to 0: $\mathbf{V}_0 = \mathbf{Y}^{-1} * \mathbf{I}_{\text{inj},0}$
- Step 2:
  Calculate the injection (compensation) currents for all the power conversion (PC) elements and adding them into the appropriate slot in the current vector to obtain $\mathbf{I}_{inj,n}$, where, n denotes the n'th iteration.
- Step 3:
  Update voltage vector: $\mathbf{V}_{n+1} = \mathbf{Y}^{-1} * \mathbf{I}_{\text{inj},n}$
- Step 4:
  Check whether the voltage error exceeds the tolerance. If not, go to Step 2.

Note, regarding the injection currents:
1. $\boldsymbol{I}_{source}$ does not change in each iteration;
2. The injection currents for a non-constant-Z load will change in each iteration.

3 Example 2: Constant-PQ Load



$$I_{k1}^{in} = I_{s1} = \frac{V_{s1}}{z_{k1}}, I_{k2}^{in} = I_{s2} = \frac{V_{s2}}{z_{k2}}, I_{k3}^{in} = I_{s3} = \frac{V_{s3}}{z_{k3}}$$

$$I_{m1}^{in} = I_{m2}^{in} = I_{m3}^{in} = 0$$

Updated in each iteration

$$\begin{cases} I_{n1}^{in} = -\left[\left(\frac{S}{3V_{n1}}\right)^* - \frac{V_{n1}}{z_{n1}}\right], \\ I_{n2}^{in} = -\left[\left(\frac{S}{3V_{n2}}\right)^* - \frac{V_{n2}}{z_{n2}}\right], \\ I_{n3}^{in} = -\left[\left(\frac{S}{3V_{n3}}\right)^* - \frac{V_{n3}}{z_{n3}}\right] \end{cases}$$

3 Example 2: Constant-PQ Load

$\mathbf{Y}^{-1}=$

																R ↓	X ↓
4.9352	0.0634	0.0003	-0.0001	0.0003	-0.0001	4.9119	0.0556	-0.0073	-0.0035	-0.0073	-0.0035	4.8812	0.055	-0.0163	-0.0068	-0.0163	-0.0068
0.0003	-0.0001	4.9352	0.0634	0.0003	-0.0001	-0.0073	-0.0035	4.9119	0.0556	-0.0073	-0.0035	-0.0163	-0.0068	4.8812	0.055	-0.0163	-0.0068
0.0003	-0.0001	0.0003	-0.0001	4.9352	0.0634	-0.0073	-0.0035	-0.0073	-0.0035	4.9119	0.0556	-0.0163	-0.0068	-0.0163	-0.0068	4.8812	0.055
4.9119	0.0556	-0.0073	-0.0035	-0.0073	-0.0035	5.5076	1.2497	0.155	0.4179	0.155	0.4179	5.4732	1.2398	0.1446	0.4084	0.1446	0.4084
-0.0073	-0.0035	4.9119	0.0556	-0.0073	-0.0035	0.155	0.4179	5.5076	1.2497	0.155	0.4179	0.1446	0.4084	5.4732	1.2398	0.1446	0.4084
-0.0073	-0.0035	-0.0073	-0.0035	4.9119	0.0556	0.155	0.4179	0.155	0.4179	5.5076	1.2497	0.1446	0.4084	0.1446	0.4084	5.4732	1.2398
4.8812	0.055	-0.0163	-0.0068	-0.0163	-0.0068	5.4732	1.2398	0.1446	0.4084	0.1446	0.4084	6.6214	2.4303	0.3614	0.8818	0.3614	0.8818
-0.0163	-0.0068	4.8812	0.055	-0.0163	-0.0068	0.1446	0.4084	5.4732	1.2398	0.1446	0.4084	0.3614	0.8818	6.6214	2.4303	0.3614	0.8818
-0.0163	-0.0068	-0.0163	-0.0068	4.8812	0.055	0.1446	0.4084	0.1446	0.4084	5.4732	1.2398	0.3614	0.8818	0.3614	0.8818	6.6214	2.4303

$$\boldsymbol{I}_{source} = \begin{bmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \end{bmatrix} = \boldsymbol{Y}_{source} * \boldsymbol{V}_{source} = \begin{bmatrix} 1625.36 + j0 \\ -812.68 - j1407.6 \\ -812.68 + j1407.6 \end{bmatrix} A$$

$$\mathbf{I}_{\mathrm{inj},0} = \begin{bmatrix} I_{k1,0}^{in} \\ I_{k2,0}^{in} \\ I_{k3,0}^{in} \\ I_{m1,0}^{in} \\ I_{m2,0}^{in} \\ I_{m3,0}^{in} \\ I_{n1,0}^{in} \\ I_{n2,0}^{in} \\ I_{n3,0}^{in} \end{bmatrix} = \begin{bmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1625.36 + j0 \\ -812.68 - j1407.6 \\ -812.68 + j1407.6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

3 Example 2: Constant-PQ Load

Matlab code for solving power flow:

```
clear;

j = sqrt(-1);

%% compute the Y matrix of each element
% Vsource
Z_k = [5+j*0, 0, 0; 0, 5+j*0, 0; 0, 0, 5+j*0];
Y_k = inv(Z_k);

% line 1
Rmatrix_1 = [0.62, 0.17, 0.17; 0.17, 0.62, 0.17; 0.17, 0.17, 0.62];
Xmatrix_1 = [1.21, 0.43, 0.43; 0.43, 1.21, 0.43; 0.43, 0.43, 1.21];

Z_km = Rmatrix_1 * 1 + j * Xmatrix_1 * 1;
Y_km = inv(Z_km);

% line 2
Rmatrix_2 = [1.19, 0.23, 0.23; 0.23, 1.19, 0.23; 0.23, 0.23, 1.19];
Xmatrix_2 = [1.21, 0.49, 0.49; 0.49, 1.21, 0.49; 0.49, 0.49, 1.21];
Z_mn = Rmatrix_2 * 1 + j * Xmatrix_2 * 1;
Y_mn = inv(Z_mn);

% load
S_load = 500 + j * 500;
S_n = [S_load/3; S_load/3; S_load/3];
V_load_mag = 13.8;
Z_LN = 1000*V_load_mag^2/conj(S_load);
Z_n = [Z_LN, 0, 0; 0, Z_LN, 0; 0, 0, Z_LN];
Y_n = inv(Z_n);

… (continued)
```

3 Example 2: Constant-PQ Load

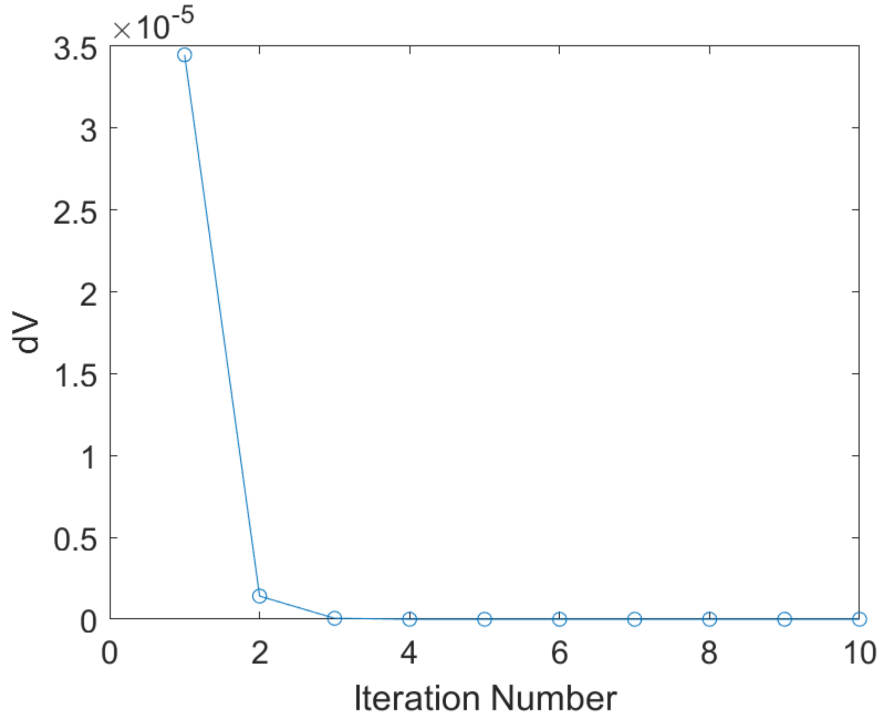
Matlab code for solving power flow:

…(continued)

```
% system Y matrix
Y = [Y_k + Y_km, -Y_km, zeros(3,3); ...
     -Y_km, Y_km + Y_mn, -Y_mn; ...
     zeros(3,3), -Y_mn, Y_mn + Y_n];

%% initialization
I_source = [(1.02*13800/3^0.5)/5; (1.02*13800/3^0.5)/5 * (-1/2-j*3^0.5/2); (1.02*13800/3^0.5)/5 * (-
1/2+j*3^0.5/2)];
I_0 = zeros(9, 1);
I_0(1:3, :) = I_source;

V_0 = Y\I_0;
V_i = V_0;
I_i = I_0;
V_last = V_0;

%% iteration
for i = 1:10
    I_load_inj =  - conj(1000*S_n ./ V_i(end-2:end, :)) + Y_n * V_i(end-2:end, :);
    I_i(end-2:end, :) = I_load_inj;
    V_i = Y\I_i;
end
```

3 Example 2: Constant-PQ Load

Results:



$$dV = \frac{||V_i| - |V_{i-1}||}{V_{nominal}}$$

Node	Real	Imaginary
Vk1	8020.78	103.35
Vk2	-3920.89	-6997.88
Vk3	-4099.90	6894.53
Vm1	7995.12	96.12
Vm2	-3914.32	-6972.04
Vm3	-4080.80	6875.92
Vn1	7959.89	100.70
Vn2	-3892.74	-6943.81
Vn3	-4067.15	6843.12

Computed voltages (volts) using Matlab

Vk1	8020.78	103.60
Vk2	-3920.67	-6998.00
Vk3	-4100.11	6894.40
Vm1	7995.13	96.29
Vm2	-3914.18	-6972.13
Vm3	-4080.95	6875.84
Vn1	7959.91	100.03
Vn2	-3893.33	-6943.50
Vn3	-4066.59	6843.47

Exported voltages (volts) from OpenDSS

Thank You!